MATEMATIKA 10

ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
II QISM

O‘rta ta’lim muassasalarining 10-sinfi va o‘rta maxsus, kasb-hunar ta’limi muassasalari o‘quvchilari uchun darslik

1-nashri

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tasdiqlagan

TOSHKENT
2017

UO‘K 51(075.32)
KBK 22.1ya721
M 54

Algebra va analiz asoslari bo‘limining mualliflari:

M.A. Mirzaahmedov, Sh.N. Ismailov, A.Q. Amanov.

Geometriya bo‘limining muallifi:

B.Q. Haydarov

Taqrizchilar:

B.Q. Beshimov – Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy Universiteti “Geometriya va topologiya” kafedrasi mudiri, fizika-matematika fanlari doktori.

M.D. Pardayeva – Respublika Ta’lim markazi direktorining o‘rinbosari.

D.E. Davletov – Nizomiy nomidagi TDPU “Matematika o‘qitish metodikasi” kafedrasi mudiri, fizika-matematika fanlari nomzodi.

R.O. Ro‘zimov – Sergeli tumani 237- umumta’lim maktabi matematika fani o‘qituvchisi.

S.R. Sumberdiyeva – Sergeli tumani 6- DIUO‘T maktabi matematika fani o‘qituvchisi.

Darslikning “Algebra va analiz asoslari” bo‘limida ishlatilgan belgilar va ularning talqini:

△ – masalani yechish (isbotlash) boshlandi

▲ – masalani yechish (isbotlash) tugadi

(!) – nazorat ishlari va test (sinov) mashqlari

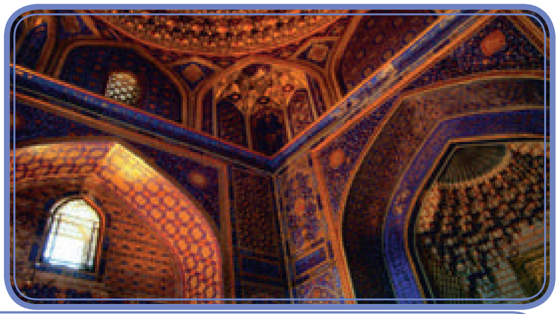
(?) – savol va topshiriqlar

(ko‘k rang) – asosiy ma’lumot

(pushti rang) – murakkabroq mashqlar

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

ISBN 978-9943-5056-7-4



III BOB

ELEMENTAR FUNKSIYALAR VA TENGLAMALAR

47-49 MUNOSABATLAR VA AKSLANTIRISHLAR. FUNKSIYA

Quyidagi jadvalda Nyu York shahrining aeroportida avtomashinalar turargohida vaqtga qarab to'lanishi lozim bo'lgan mablag' miqdorlari keltirilgan:

Ravshanki, to'lanadigan mablag' qiymati vaqt davomiyligiga bevosita bog'liq.

Vaqt (t)	Qiymati
0 – 1 soat	$5,00
1 – 2 soat	$9,00
2 – 3 soat	$11,00
3 – 6 soat	$13,00
6 – 9 soat	$18,00
9 – 12 soat	$22,00
12 – 24 soat	$28,00

Bu jadvalga qarab quyidagi savolga javob beraylik:

Avtomashinaning aynan bir soat turishi uchun qancha pul sarflanadi?

5 AQSh dollarimi, 9 AQSh dollarimi yoki 11 AQSh dollarimi?

Noqulay vaziyatga tushmaslik va muammoni aniqlashtirish uchun biz jadvaldagi ma'lumotlarni grafik ko'rinishiga keltiramiz. Jadvaldagi "2 – 3 soat" yozuv "2 soatdan ortiq ammo 3 soatdan ortiqmas vaqt", ya'ni $2 < t \leqslant 3$ oraliq deb tushuniladi. U holda quyidagi jadvalni hosil qilamiz:

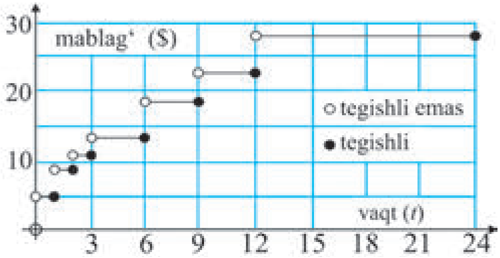
Vaqt (t)	**Qiymati**
$0 < t \leqslant 1$ soat	$5,00
$1 < t \leqslant 2$ soat	$9,00
$2 < t \leqslant 3$ soat	$11,00
$3 < t \leqslant 6$ soat	$13,00
$6 < t \leqslant 9$ soat	$18,00
$9 < t \leqslant 12$ soat	$22,00
$12 < t \leqslant 24$ soat	$28,00

Matematika tilida mazkur jadval ikkita o'zgaruvchi (*vaqt* va to'lanadigan *mablag' miqdori*) orasidagi **munosabatga** misol bo'la oladi.

Munosabat tartiblangan juftliklar to'plami sifatida talqin qilinishi mumkin, masalan,

$\{(1, 5), (-2, 3), (4, 3), (1, 6)\}$.

Avtomashinalar turargohida $0 < t \leqslant 24$ oraliqdagi t vaqtga qarab to'lanishi lozim bo'lgan mablag' o'zgarishi quyidagicha tasvirlanadi:



Gorizontal o‘qdagi o‘zgaruvchining qabul qiladigan qiymatlar to‘plami munosabatning *aniqlanish sohasi* deyiladi.

Masalan, $\{t|0<t\leqslant 24\}$ to‘plam yuqoridagi *vaqt* va to‘lanadigan *mablag‘ miqdori* orasidagi munosabatning, $\{-2, 1, 4\}$ to‘plam esa {(1, 5), (− 2, 3), (4, 3), (1, 6)} munosabatning aniqlanish sohalari bo‘ladi.

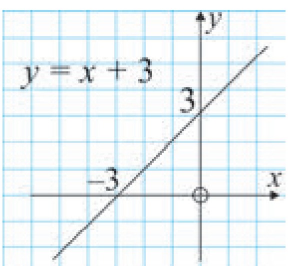
Vertikal o‘qdagi o‘zgaruvchining qabul qiladigan qiymatlar to‘plami munosabatning *qiymatlar to‘plami* deyiladi.

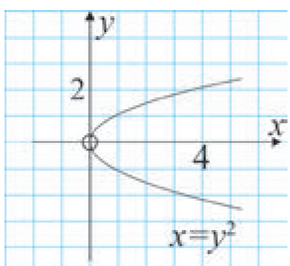
Masalan, {5, 9, 11, 13, 18, 22, 28} to‘plam yuqoridagi *vaqt* va to‘lanadigan *mablag‘ miqdori* orasidagi munosabatning, {3, 5, 6} to‘plam esa {(1, 5), (−2, 3), (4, 3), (1, 6)} munosabatning qiymatlar to‘plamlari bo‘ladi.

Endi munosabatga aniqroq ta’rif beraylik. Dekart koordinatalar tekisligida berilgan nuqtalar to‘plami **munosabat** deyiladi. Ko‘pincha munosabat x, y **o‘zgaruvchilar** qatnashgan tenglama ko‘rinishida beriladi.

Masalan, $y=x+3$, $x=y^2$ tenglamalarning har biri munosabatni aniqlaydi.

Bu tenglamalarning har biri Dekart koordinatalar tekisligida nuqtalar to‘plamini hosil qiladi.

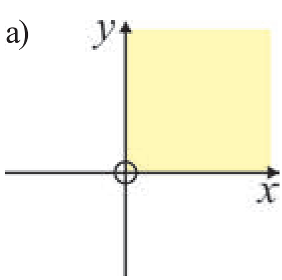




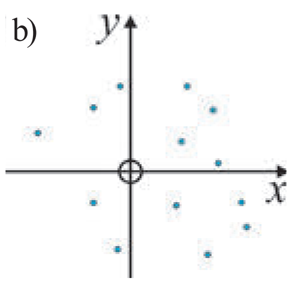
Ayrim munosabatlarni tenglamalar yordamida yozib bo‘lmaydi.

Masalan, $x>0$, $y>0$ shartni qanoatlantiradigan (x, y) nuqtalar to‘plami (koordinatalar tekisligining birinchi choragi, *a*-rasm) yoki *b*- rasmdagi nuqtalar

a)



b)



to‘plamini tenglamalar yordamida yozib bo‘lmaydi.

Agar munosabatda birinchi koordinatasi teng bo‘lgan ikkita turli nuqta mavjud bo‘lmasa, bu munosabat **akslantirish** yoki **funksiya** deyiladi.

Demak, funksiya – munosabatning maxsus turi ekan.

Berilgan munosabat funksiya ekanligini tekshirishning ikki usulini keltiramiz.

Algebraik usul

Bu usul munosabat tenglama yordamida berilgan hollarda qo‘llaniladi. Bunda berilgan tenglamaga x va y ning ixtiyoriy qiymatini qo‘yganda x ning har bir qiymati uchun y ning yagona qiymati hosil bo‘lsa, bunday munosabat funksiya bo‘ladi.

Masalan, $y=3x-2$ tenglamaga x ning ixtiyoriy qiymatini qo‘ysak, y ning yagona qiymati hosil bo‘ladi. Demak, bu tenglama yordamida aniqlangan munosabat funksiya bo‘ladi.

Shu bilan birga $x=y^2$ tenglama bilan aniqlangan munosabat funksiya bo‘lmaydi, chunki, masalan, $x=4$ qiymatini qo‘ysak, ikkita $y=\pm 2$ qiymat hosil bo‘ladi.

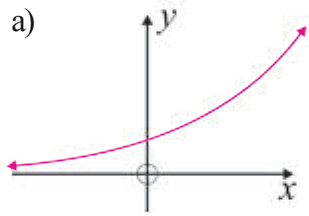
Grafik usul

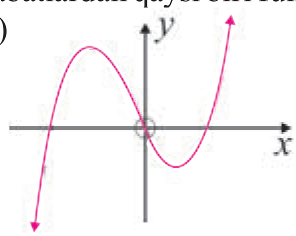
Munosabat Dekart koordinatalar sistemasida to‘plam ko‘rinishida berilgan bo‘lsin. Agar biz barcha mumkin bo‘lgan vertikal to‘g‘ri chiziqlarni chizsak, bu to‘g‘ri chiziqlardan ixtiyoriysining berilgan munosabat bilan kesishish nuqtalari soni bittadan oshmasa, u holda bu munosabat funksiya bo‘ladi. Aksincha, agar qandaydir vertikal to‘g‘ri chiziqning berilgan munosabat bilan kesishish nuqtalari soni bittadan ko‘p bo‘lsa, u holda munosabat funksiya bo‘lmaydi.

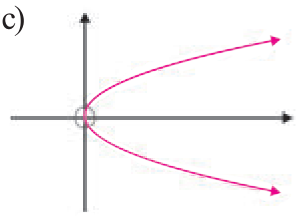
Bunda biz quyidagilarga shartli ravishda kelishamiz:

- Agar chiziqda kichik oq rangdagi doiracha belgilangan bo‘lsa (—o—), bunday nuqta chiziqqa tegishli emas.
- Agar chiziqda kichik qora rangdagi doiracha belgilangan bo‘lsa (—●—), bu nuqta chiziqqa tegishli.
- ⟶ ko‘rinishdagi (strelka) o‘q chiziq shu yo‘nalishda cheksiz davom ettirilishi mumkinligini bildiradi.

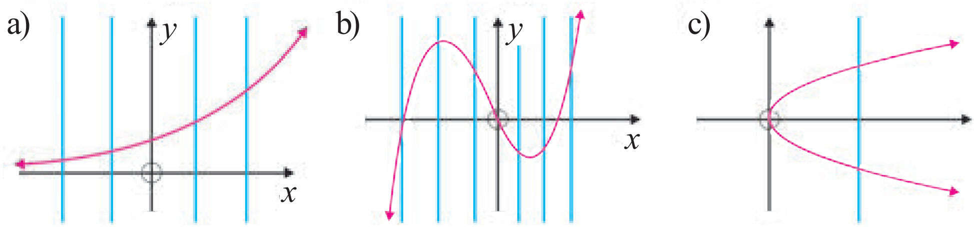
1- misol. Quyidagi munosabatlardan qaysi biri funksiya bo‘lishini tekshiraylik:

a)


b)


c)


△ Vertikal to‘g‘ri chiziqlarni chizib, shunday xulosaga kelamiz:



a) va b) munosabatlardan har biri funksiya bo‘ladi (chunki ixtiyoriy vertikal to‘g‘ri chiziq u bilan eng ko‘pi bitta nuqtada kesishadi), c) munosabat esa funksiya emas, chunki uni ikkita nuqtada kesuvchi vertikal to‘g‘ri chiziq mavjud. ▲

Hisoblash uskunasi (moslamasi) quyidagi algoritm bo‘yicha ishlasin:

1- qadam. Biror son kiritilmoqda.

2- qadam. Kiritilgan son 2 ga ko‘patirilmoqda.

3- qadam. Natijaga 3 qo‘shilmoqda.

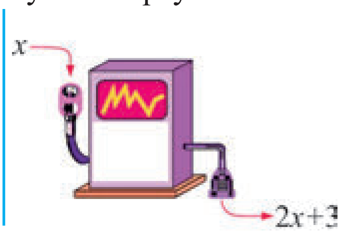
Masalan, uskunaga 4 soni kiritilsa, natijada $4 \cdot 2+3=11$ soni hosil bo‘ladi.

Xuddi shunday uskunaga (-4) soni kiritilsa, natijada $2 \cdot (-4)+3=-5$ soni hosil bo‘ladi.

Umumiy holda, uskunaga x soni kiritilsa, natijada yagona $2x+3$ soni hosil bo‘ladi.

Ko‘rinib turibdiki, uskunaga qandaydir x son kiritilsa, natijada yagona $2x+3$ qiymat hosil bo‘ladi.

Demak, mazkur uskuna ishlaydigan algoritm funksiyani aniqlaydi.



Bu holat $f{:}x \mapsto 2x+3$, $f(x)=2x+3$ yoki $y=2x+3$ kabi yoziladi.

Agar $f(x)=2x+3$ bo‘lsa, uning -4 soniga mos qiymati $f(-4)=2(-4)+3=-5$ kabi topiladi.

Umumiy holda, $f(x)$ − funksiyaning berilgan x sondagi *qiymati* deb yuritiladi va mazkur munosabat $y=f(x)$ kabi yoziladi.

2- misol. Agar $f{:}x \mapsto 2x^2-3x$ bo‘lsa: a) $f(5)$; b) $f(-4)$ qiymatlarni toping.

△ $f(x)=2x^2-3x$ munosabatga $x=5$ va $x=-4$ sonlarni qo‘yib, ularga mos qiymatlarni topamiz: a) $f(5)=2 \cdot 5^2-3 \cdot 5=2 \cdot 25-15=35$;

b) $f(-4)=2 \cdot (-4)^2-3 \cdot (-4)=2 \cdot 16+12=44$. ▲

3- misol. Agar $f(x)=5-x-x^2$ bo‘lsa: a) $f(-x)$; b) $f(x+2)$ qiymatlarni toping va natijalarni soddalashtiring.

△ $f(x)=5-x-x^2$ funksiyaga x o‘rniga $-x$ va $x+2$ qiymatlarni qo‘yib, ularga mos qiymatlarni topamiz:

a) $f(-x)=5-(-x)-(-x)^2=5+x-x^2$;

b) $f(x+2)=5-(x+2)-(x+2)^2=5-x-2-(x^2+4x+4)=3-x-x^2-4x-4=$
$=-x^2-5x-1.$ ▲

Savol va topshiriqlar

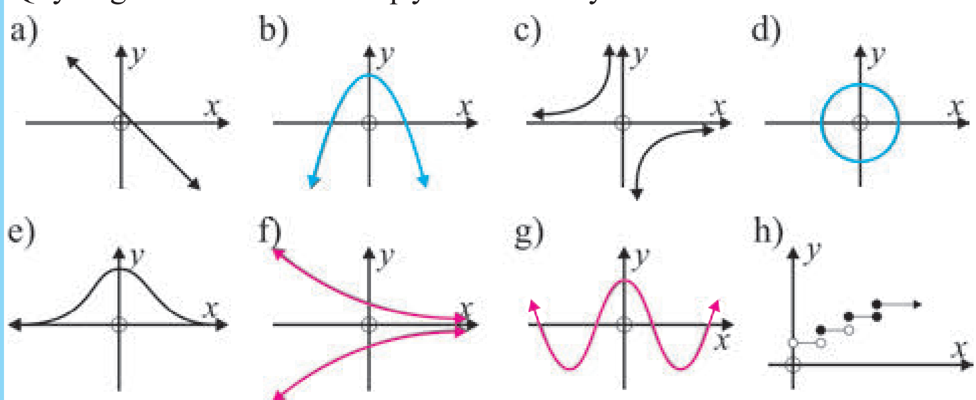
1. Munosabatga misollar keltiring.
2. Akslantirish yoki funksiyaga ta'rif bering.
3. Funksiyaninig aniqlanish sohasini tushuntiring.
4. Funksiyaninig qiymatlar sohasini tushuntiring.

Mashqlar

73. Quyidagi munosabatlardan qaysilari funksiya bo'ladi:

a) $\{(1, 3), (2, 3), (3, 5), (4, 6)\}$;
b) $\{(1, 3), (3, 2), (1, 7), (-1, 4)\}$;
c) $\{(2, -1), (2, 0), (2, 3), (2, 11)\}$;
d) $\{(7, 6), (5, 6), (3, 6), (-4, 6)\}$;
e) $\{(0, 0), (1, 0), (3, 0), (5, 0)\}$;
f) $\{(0, 0), (0, -2), (0, 2), (0, 4)\}$?

74. Quyidagi munosabatlardan qaysilari funksiya bo'ladi?



75. Dekart koordinatalar tekisligida berilgan har qanday to'g'ri chiziq funksiya bo'ladimi? Javobingizni asoslang.

76. $x^2+y^2=9$ tenglama yordamida berilgan munosabat funksiya bo'ladimi?

77. Agar $f:x\mapsto 3x+2$ bo'lsa, quyidagi qiymatlarni toping:

A) $f(0)$; B) $f(2)$; C) $f(-1)$; D) $f(-5)$; E) $f\left(-\frac{1}{3}\right)$.

78. Agar $f:x\mapsto 3x-x^2+2$ bo'lsa, quyidagi qiymatlarni toping:

A) $f(0)$; B) $f(3)$; C) $f(-3)$; D) $f(-7)$; E) $f\left(\frac{2}{3}\right)$.

79. Agar $g:x\mapsto x-\frac{4}{x}$ bo'lsa, quyidagi qiymatlarni toping:

A) $g(1)$; B) $g(4)$; C) $g(-1)$; D) $g(-4)$; E) $g\left(-\frac{1}{2}\right)$.

80. Agar $f(x)=7-3x$ bo'lsa, quyidagi qiymatlarni toping va natijani mumkin bo'lsa soddalashtiring.

a) $f(a)$; | b) $f(-a)$; | c) $f(a+3)$; | d) $f(b-1)$; | e) $f(x+2)$; | f) $f(x+h)$.

81. Agar $F(x)=2x^2+3x-1$ bo'lsa, quyidagi qiymatlarni toping va natijani soddalashtiring.

a) $F(x+4)$; | b) $F(2-x)$; | c) $F(-x)$; | d) $F(x^2)$; | e) $F(x^2-1)$; | f) $F(x+h)$.

82. $G(x)=\dfrac{2x+3}{x-4}$ funksiya uchun:

a) **I** $G(2)$ **II** $G(0)$ **III** $G\left(-\dfrac{1}{2}\right)$ larni toping;

b) Qanday x larda $G(x)$ mavjud emas?

c) $G(x+2)$ ni toping va soddalashtiring;

d) x ning $G(x)=-3$ bo'ladigan qiymatini toping.

83. Funksiya f harfi bilan belgilangan bo'lsin. f va $f(x)$ belgilarning ma'nolari orasida qanday farq bor?

84. Eskirish natijasida nusxa ko'paytirish uskunasining t yildan so'ng narxi $V(t)=9650-860t$ qonuniyat bo'yicha o'zgaradi.

a) $V(4)$ ni toping va uning ma'nosini tushuntiring;

b) $V(t)=5780$ bo'lganda t ni toping. Vaziyatni tushuntiring;

c) Uskuna qaysi narxda sotib olingan?

85. Bitta koordinatalar tekisligida $f(2)=1$, $f(5)=3$ bo'ladigan uchta turli funksiya grafiklarini chizing.

86. $f(2)=1$ va $f(-3)=11$ bo'ladigan $f(x)=ax+b$ chiziqli funksiyani toping.

87. $f(x)=ax+\dfrac{b}{x}$, $f(1)=1$, $f(2)=5$ bo'lsa, a, b larni toping.

88. $T(0)=-4$, $T(1)=-2$, $T(2)=6$ bo'ladigan $T(x)=ax^2+bx+c$ kvadrat funksiyani toping.

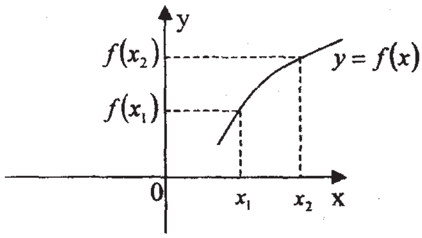
89. $f(x)=2^x$ bo'lsa, $f(a)f(b)=f(a+b)$ tenglikni isbotlang.

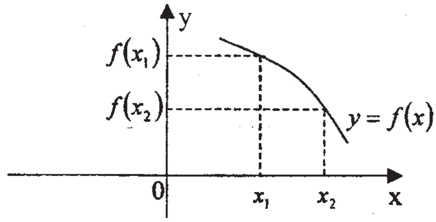
50-51 ELEMENTAR FUNKSIYALARNING MONOTONLIGI, ENG KATTA VA ENG KICHIK QIYMATLARI HAQIDA TUSHUNCHA

Funksiyaning monotonligi

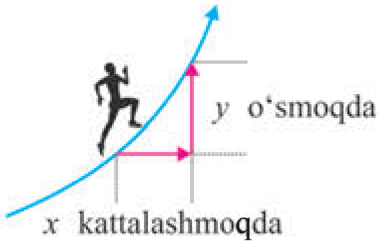
Agar $x_1<x_2$ tengsizlikni qanoatlantiruvchi barcha $x_1, x_2\in I$ uchun $f(x_1)<f(x_2)$ tengsizlik o'rinli bo'lsa, I oraliqda $y=f(x)$ funksiya *o'suvchi* deyiladi.

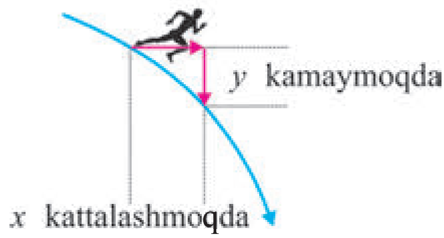
Agar $x_1<x_2$ tengsizlikni qanoatlantiruvchi barcha $x_1, x_2\in I$ uchun $f(x_2)<f(x_1)$ tengsizlik o'rinli bo'lsa, I oraliqda $y=f(x)$ funksiya *kamayuvchi* deyiladi.



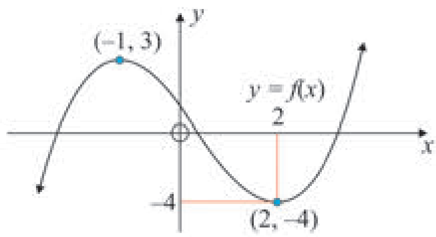


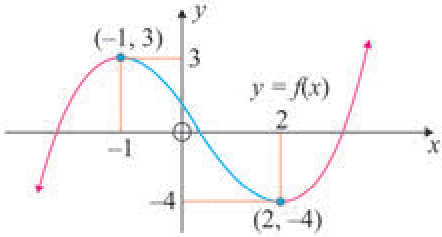
Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga “harakat” qilsak, ordinatalar ortadi; funksiya kamayuvchi bo‘lsa, ordinatalar kamayadi.





1- misol. Funksiyaning o‘sish va kamayish oraliqlarini toping:





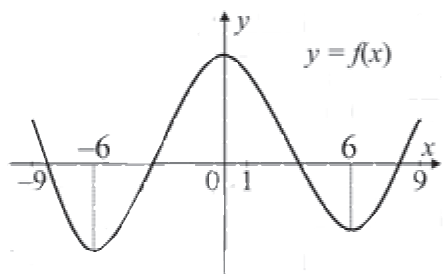
△Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar o‘sadi (grafikda qizil rangda ajratilgan). Demak, funksiya $x \leqslant -1$ va $x \geqslant 2$ oraliqlarda o‘sadi. Javobni $(-\infty,-1] \cup [2,+\infty)$ ko‘rinishda ham yozsa bo‘ladi.

Xuddi shunday, agar funksiya kamayuvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kamayadi (grafikda ko‘k rangda ajratilgan). Demak, funksiya $-1 \leqslant x \leqslant 2$ oraliqlarda kamayadi. ▲

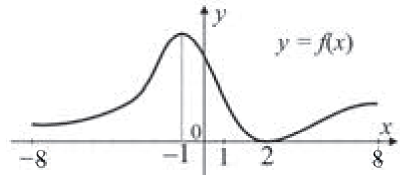
2- misol. Funksiya qaysi oraliqlarda o‘sadi?

△Bu funksiya $[-9;\ 9]$ oraliqda berilgan.

Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kattalashadi. Demak, funksiya $[-6;0]$ va $[6;9]$ oraliqlarda o‘sadi. Javobni $[-6;0] \cup [6;9]$ ko‘rinishda ham yozsa bo‘ladi. ▲

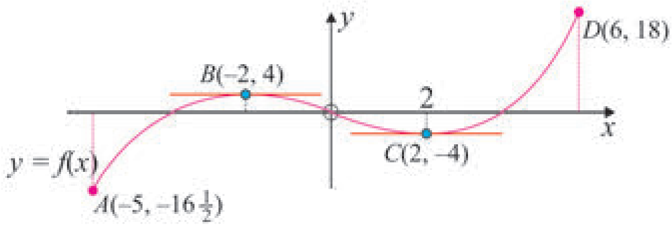


3- misol. Funksiya qaysi oraliqlarda kamayadi?



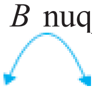
Agar funksiya kamayuvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kichiklashadi. Demak, funksiya $[-1;\ 2]$ oraliqda kamayadi.

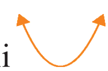
Funksiyaning eng katta va eng kichik qiymatlari haqida tushuncha beramiz. $-5 \leqslant x \leqslant 6$ oraliqda aniqlangan funksiya grafigini qaraylik.



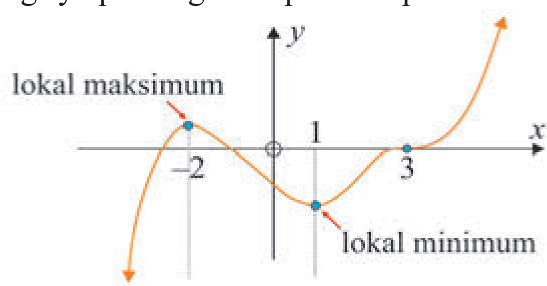
A nuqtaning ordinatasi boshqa nuqtalar ordinatalaridan kichik bo‘lgani sababli shu nuqta **global minimum** nuqtasi deyiladi. Funksiyaning unga mos bo‘lgan qiymati **funksiyaning eng kichik qiymati** deyiladi. Bizning misolimizda funksiyaning eng kichik qiymati −16,5 ga teng.

Xuddi shunday, *D* nuqtaning ordinatasi boshqa nuqtalar ordinatalaridan katta bo‘lgani sababli shu nuqta **global maksimum** nuqtasi deyiladi. Funksiyaning unga mos bo‘lgan qiymati **funksiyaning eng katta qiymati** deyiladi. Bizning misolimizda funksiyaning eng katta qiymati 18 ga teng.

Endi *B* nuqtaga e’tibor beraylik. Grafikning unga yaqin bo‘lgan nuqtalari to‘plami  shaklga ega. Bunday xossaga ega bo‘lgan nuqta **lokal maksimum** nuqtasi deyiladi.

Huddi shunday, grafikning *C* nuqtaga yaqin bo‘lgan nuqtalari to‘plami  shaklga ega. Bunday xossaga ega bo‘lgan nuqta **lokal minimum** nuqtasi deyiladi.

Faqat lokal minimum va lokal maksimumga ega bo‘lgan funksiyaga misol keltiraylik:



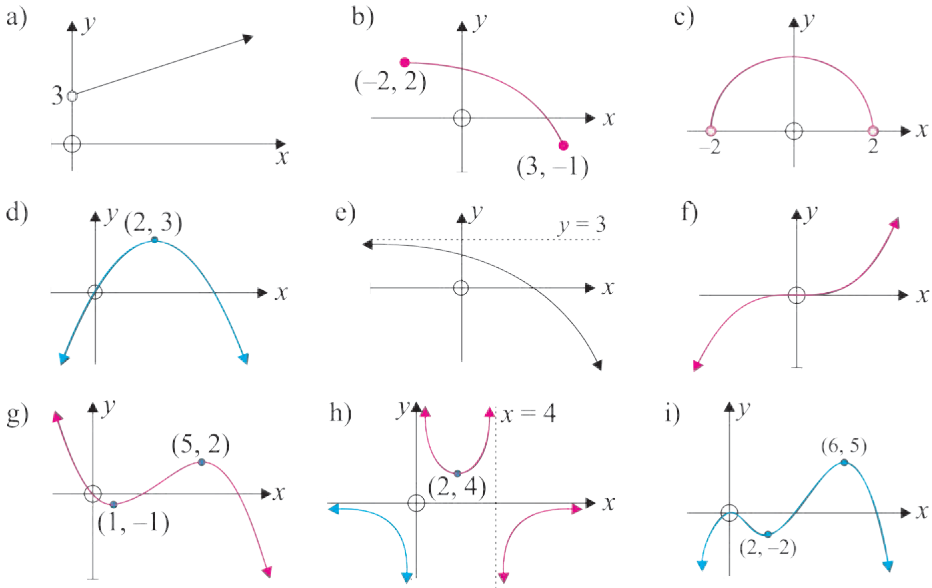
Savol va topshiriqlar

1. Oraliqda o‘suvchi funksiyaga ta’rif bering.
2. Oraliqda kamayuvchi funksiyaga ta’rif bering.

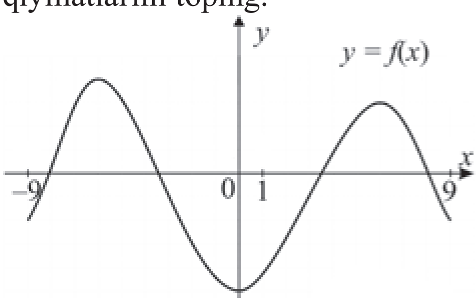
3. Chizmaga qarab funksiyaning o‘sishi qanday aniqlanadi?
4. Chizmaga qarab funksiyaning kamayishi qanday aniqlanadi?

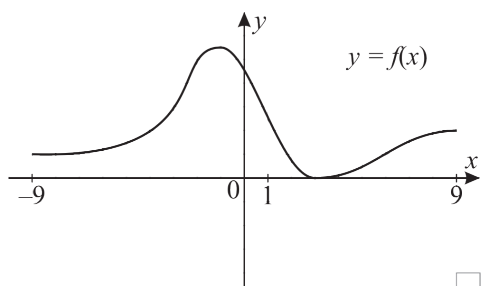
Mashqlar

90. Grafigi berilgan funksiya uchun: **I)** o‘sish; **II)** kamayish oraliqlarni toping. Agar mumkin bo‘lsa, ularning lokal maksimumini va lokal minimumini, eng katta va eng kichik qiymatlarini toping:



91. [–9; 9] oraliqda berilgan funksiya qaysi oraliqlarda o‘sadi?
Qaysi oraliqlarda kamayadi?
Uning lokal maksimumini va lokal minimumini, eng katta va eng kichik qiymatlarini toping:



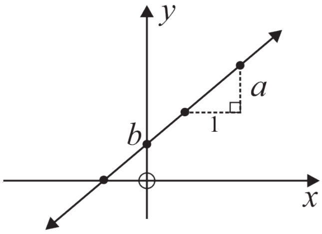


52-54 CHIZIQLI VA KVADRATIK MODELLAR

Chiziqli funksiya

$f(x)=ax+b$ ko'rinishdagi funksiya chiziqli deyiladi, bu yerda x, y – o'zgaruvchilar, a, b – berilgan sonlar, $a\neq 0$.

Chiziqli funksiya grafigi koordinata tekisligida to'g'ri chiziq bo'lib, bunda a son burchak koeffitsiyenti deyiladi.



Quyida biz chiziqli funksiya tatbiqlarini keltiramiz.

1- misol. Tennis kortini ijaraga olish narxi $C(h)=5h+8$ (AQSh dollari) formula bilan aniqlangan, bu yerda h – ijara vaqti (soatda). 4 soat va 10 soat uchun ijaraga qancha mablag' sarflanadi?

△ $C(h)=5h+8$ formuladan foydalanib, $C(4)=5\cdot 4+8=20+8=28$ va $C(10)=5\cdot 10+8=50+8=58$ ekanligini topamiz. Demak, 4 soatga 28 AQSh dollari, 10 soatga esa 58 AQSh dollari mablag' sarflanadi. ▲

2- misol. Nyu Yorkda taksi passajir olish uchun to'xtashga 3 AQSh dollari va 30 sent, har kilometrga esa 1 AQSh dollari va 75 sent oladi.

a) Jadvalni daftaringizga ko'chirib oling va uni to'ldiring:

d – masofa (km)	0	2	4	6	8	10
C – mablag' (\$)						

b) C va d orasidagi bog'lanishni grafik ko'rinishda ifodalang;

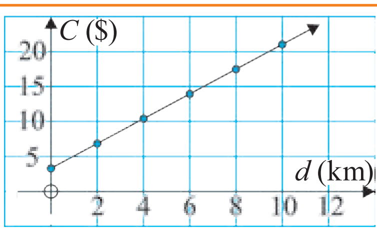
c) C (d) funksiyaning algebraik ko'rinishini–formulasini yozing;

d) 9,4 km yurish uchun qancha mablag' sarflanadi?

△ a) 3,3 AQSh dollariga ketma-ket $2\cdot 1{,}75=3{,}5$ AQSh dollarini qo'shib, kataklarni to'ldiramiz:

d – masofa (km)	0	2	4	6	8	10
C – mablag' (\$)	3,30	6,80	10,30	13,80	17,30	20,80

b)



Bu – chiziqli funksiya.

c) Burchak koeffitsiyentini topamiz:

$$a=\frac{20{,}80-17{,}30}{10-8}=1{,}75.$$

Demak, $C(d)=1{,}75d+3{,}3$.

d) $C(9{,}4)=1{,}75\cdot 9{,}4+3{,}3=19{,}75$.

Demak, 19,75 AQSh dollari sarflanadi. ▲

Kvadrat funksiya

$y=ax^2+bx+c$ ko'rinishdagi funksiya kvadrat funksiya deyiladi, bu yerda x, y – o'zgaruvchilar, a, b, c – berilgan sonlar, $a\neq 0$.

$y=2x^2+4x-5$ funksiyaning a) $x=0$; b) $x=3$ nuqtalardagi qiymatini topaylik.

a) $x=0$ bo'lsin. U holda $y=2\cdot 0^2+4\cdot 0-5=0+0-5=-5$.

b) $x=3$ bo'lsin. U holda $y=2\cdot 3^2+4\cdot 3-5=18+12-5=25$.

3- misol. Tosh otilganda t sekundda uning yerga nisbatan balandligi $h(t)=-5t^2+30t+2$ funksiya yordamida aniqlanadi.

a) $t=3$ bo'lganda tosh yerdan qancha balandlikda bo'ladi?

b) Tosh qanday balandlikdan turib otildi?

c) Qaysi vaqtda toshning balandligi 27 m bo'ladi?

△ a) $h(3)=-5\cdot 3^2+30\cdot 3+2=-45+90+2=47$. Demak, otilgan tosh $t=3$ sekunddan so'ng 47 m balandlikda bo'ladi.

b) tosh $t=0$ bo'lganda otilgani bois, $h(0)=-5\cdot 0^2+30\cdot 0+2=2$. Demak, tosh 2 metr balandlikdan otilgan.

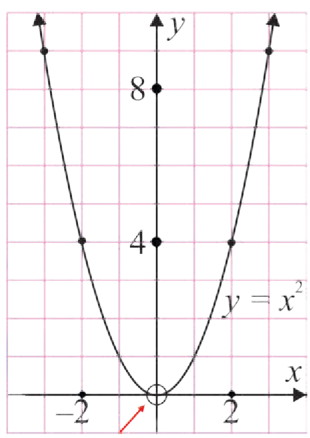
c) Tosh yerdan 27 m balandlikda bo'lsa, $h(t)=27$ bo'ladi, ya'ni $-5t^2+30t+2=27$. Bu tenglamani yechamiz: $-5t^2+30t-25=0$, $t^2-6t+5=0$, $t_1=1$, $t_2=5$. Demak, tosh 27 m balanlikda 1 sekunddan so'ng (tepaga ko'tarilayotganda) va 5 sekunddan so'ng (pastga tushayotganda) bo'ladi. ▲

Kvadrat funksiya grafigi

$f(x)=x^2$ funksiyani qaraylik. Uning ba'zi nuqtalardagi qiymatlari jadvalini tuzamiz:

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

Shu jadvaldagi (x, y) nuqtalarni koordinata tekisligida yasab, ularni silliq chiziq bilan tutashtirib, ushbu grafikni hosil qilamiz:



Hosil bo'lgan shakl **parabola** deb ataladi. Ko'rinib turibdiki, parabola tarmoqlari yuqoriga yo'nalgan bo'lib, u ordinata o'qiga nisbatan simmetrik bo'lgan egri chiziqdir.

(0; 0) nuqta $y=x^2$ **parabolaning uchi** deyiladi.

4-misol. $y=x^2-2x-5$ kvadrat funksiya grafigini yasang.

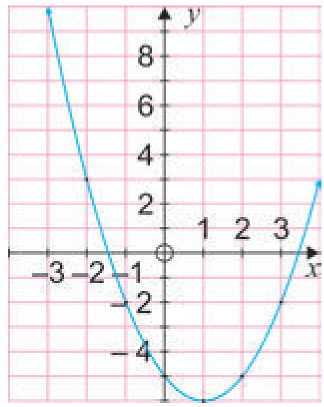
△ Funksiyaning bitta nuqtadagi, masalan $x=-3$ nuqtasidagi qiymatini topaylik:

$f(-3)=(-3)^2-2(-3)-5=9+6-5=10$.

Funksiyaning bir nechta nuqtadagi qiymatini topib, jadvalni tuzamiz:

x	-3	-2	-1	0	1	2	3
y	10	3	-2	-5	-6	-5	-2

$(x,\ y)$ nuqtalarni koordinata tekisligida yasab, ularni silliq chiziq bilan tutashtirib, berilgan kvadrat funksiya grafigini hosil qilamiz:



Hosil bo‘lgan grafik ham parabola shaklida. Uning tarmoqlari esa yuqoriga yo‘nalgan. ▲

Ixtiyoriy $y=ax^2+bx+c$ parabolaning ordinatalar o‘qi – Oy o‘qi bilan kesishish nuqtasini topamiz:

$x=0,\ \ y=a\cdot0^2+b\cdot0+c=0+0+c=c.$

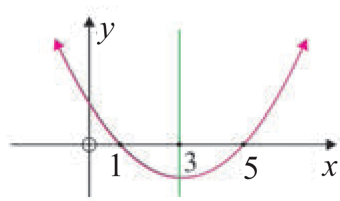
Demak, parabola $(0,\ c)$ nuqtada ordinatalar o‘qi bilan kesishadi.

$y=ax^2+bx+c$ parabolaning abssissalar o‘qi bilan kesishish nuqtalarini topish uchun $ax^2+bx+c=0$ kvadrat tenglamaning yechimlarini topish kifoya.

Masalan, $y=x^2-2x-15$ parabolaning abssissalar o‘qi bilan kesishish nuqtalarini topamiz. $x^2-2x-15=0$ deb, bu kvadrat tenglamani yechamiz. Uning yechimlari $x=-3$ va $x=5$ bo‘ladi. Demak, $y=x^2-2x-15$ parabola abssissalar o‘qi bilan $(-3,\ 0)$, $(5,\ 0)$ nuqtalarda kesishishadi. $y=ax^2+bx+c$ parabola uchun $x=h$ ko‘rinishdagi vertikal to‘g‘ri chiziq uning *simmetriya* o‘qi bo‘ladi.

Agar $y=ax^2+bx+c$ parabola abssissalar o‘qi bilan kesishsa, h son parabolaning Ox o‘qi bilan kesishish nuqtalari abssissalarining o‘rta arifmetigiga teng bo‘ladi.

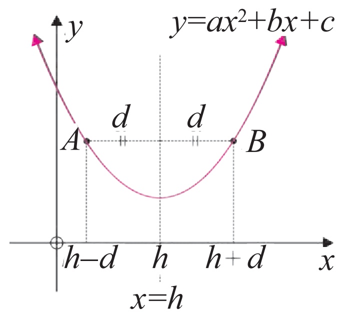
5- misol. Rasmdagi parabolaning simmetriya o‘qini toping.



△ Agar parabola abssissalar o‘qi bilan $(1,\ 0)$ va $(5,\ 0)$ nuqtalarda kesishsa, $x=\dfrac{5+1}{2}=3$ – simmetriya o‘qi bo‘ladi. ▲

Agar $y=ax^2+bx+c$ parabola abssissalar o‘qi bilan kesishmasa, h sonni boshqa usulda ham topsa bo‘ladi.

Ko‘rinib turibdiki, abssissalari $h-d$ va $h+d$ bo‘lgan $A,\ B$ nuqtalar bir xil ordinatalarga ega, ya’ni $f(h-d)=f(h+d)$, demak, A va B nuqtalar $x=h$ o‘qqa nisbatan simmetrik nuqtalardir.



Bu shartdan foydalanib quyidagi tenglikdan h ni topamiz:

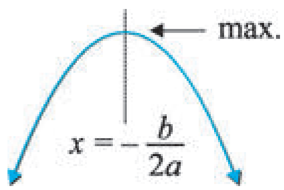
$a(h-d)^2+b(h-d)+c=a(h+d)^2+b(h+d)+c$ yoki

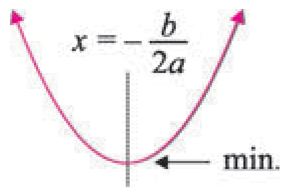
$a(h^2-2hd+d^2)+bh-bd=a(h^2-2hd+d^2)+bh+bd,$ yoki $-4ahd=2bd$, bundan $h=\frac{-b}{2a}$. Demak, simmetriya o'qi $x=\frac{-b}{2a}$ ekan.

Xulosa. $y=ax^2+bx+c$ parabolaning simmetriya o'qi $x=\frac{-b}{2a}$ bo'ladi. Parabolaning o'z-o'ziga simmetrik bo'lgan nuqtasi parabolaning uchi deyiladi.

Parabola uchining koordinatalari $x=\frac{-b}{2a}$, $y=y(\frac{-b}{2a})=\frac{-(b^2-4ac)}{4a}$. Parabola o'qi $(-\frac{b}{2a},0)$ nuqtadan Oy o'qiga paralel bo'lib o'tadi.

Ravshanki, $a<0$ bo'lganda parabola shakli  kabi bo'lib, uning uchi $y=ax^2+bx+c$ kvadrat funksiyaning maksimum nuqtasi, $a>0$ bo'lganda parabola shakli  kabi bo'lib, uning uchi kvadrat funksiyaning minimum nuqtasi bo'ladi.





6- misol. $y=3x^2+4x-5$ parabolaning simmetriya o'qini toping.

△ $y=3x^2+4x-5$ uchun $a=3,\ b=4$.

Demak, $x=\frac{-b}{2a}=\frac{-4}{2\cdot 3}=-\frac{2}{3}$, ya'ni $x=-\frac{2}{3}$ − simmetriya o'qi. ▲

7- misol. $f(x)=x^2+6x+4$ parabolaning uchini toping.

△ $a=1,\ b=6.\quad x=\frac{-b}{2a}=\frac{-6}{2\cdot 1}=-3$.

Demak, parabola uchining abssissasi $x=-3$,

ordinatasi esa: $y=f(-3)=(-3)^2+6(-3)+4=9-18+4=-5$.

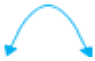
Shuning uchun, parabola uchi $(-3,-5)$ koordinatalarga ega. ▲

8- misol. Sportchi to'pni yuqoriga otdi, bunda to'pning t sekunddan keyingi balandligi $H(t)=30t-5t^2$ metr bo'ldi, $t\geqslant 0$.

a) Necha sekundda to'p eng yuqori nuqtaga yetadi?

b) Eng yuqori nuqta yerdan qancha balandlikda bo'ladi?

c) To'p necha sekunddan keyin yerga tushadi?

a) $H(t)=30t-5t^2$ uchun $a<0$, $a=-5$. Shuning uchun bu parabola quyidagi shaklda bo‘ladi: . $t=\dfrac{-b}{2a}=\dfrac{-30}{2\cdot(-5)}=3$ sekundda maksimumga erishiladi.

Ya’ni eng yuqori nuqtaga to‘p 3 sekundda ko‘tariladi.

b) Maksimal balandlikni topamiz:

$H(3)=30\cdot3-5\cdot3^2=90-45=45$, ya’ni eng yuqori nuqta yerdan 45 metr balandlikda bo‘ladi.

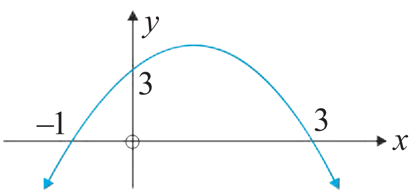
c) $H(t)=0$ bo‘lsa, to‘p yerga tushadi. Shu tenglamani yechamiz:

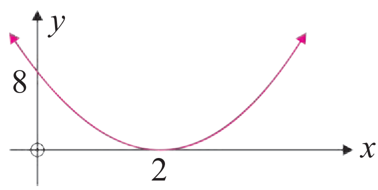
$30t-5t^2=0$, $5t^2-30t=0$, $5t(t-6)=0$. Bundan $t_1=0$ yoki $t_2=6$.

Demak, 6 sekunddan keyin to‘p yerga tushadi.

Quyida biz parabola shakliga qarab kvadrat funksiya formulasini topishga doir misollar keltiramiz.

9- misol. Berilgan parabolalarga qarab kvadrat funksiya formulasini yozing:

a) 

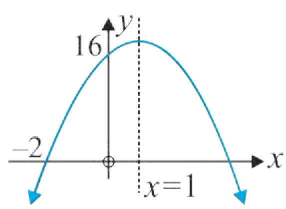
b) 

a) Parabola tarmoqlari pastga qaragan, u abssissalar o‘qi bilan -1 va 3 nuqtalarda kesishadi. Shuning uchun $y=a(x+1)(x-3)$, $a<0$. $x=0$ da $y=3$ shartdan $a=-1$ ni topamiz.

Demak, kvadrat funksiya $y=-(x+1)(x-3)=-x^2+2x+3$ formula bilan ifodalanadi.

b) Parabola tarmoqlari yuqoriga qaragan, u abssissalar o‘qiga $x=2$ nuqtada urinadi. Shuning uchun $y=a(x-2)^2$, $a>0$. $x=0$ da $y=8$ shartdan $a=2$ ni topamiz. Demak, kvadrat funksiya $y=2(x-2)^2$ formula bilan beriladi.

10- misol. Berilgan parabolaga qarab kvadrat funksiya formulasini yozing.



$x=1$ – simmetriya o‘qi bo‘lgani sababli, abssissalar o‘qi bilan ikkinchi kesishish nuqtasi $x=4$ bo‘ladi. Demak, $y=a(x+2)(x-4)$. Bundan $x=0$, $y=16$. Shuning uchun $16=a(0+2)(0-4)$. Bu yerdan $a=-2$ yoki $y=-2(x+2)\cdot(x-4)=-2x^2+4x+16$.

Savol va topshiriqlar

1. Chiziqli funksiya nima?
2. Chiziqli funksiyaning burchak koeffitsiyenti nima?
3. Kvadrat funksiya nima?

4. Kvadrat funksiyaning uchi qanday topiladi?
5. Qachon kvadrat funksiya maksimumga ega bo‘ladi?
6. Qachon kvadrat funksiya minimumga ega bo‘ladi?

Mashqlar

92. Eskirishi natijasida avtomashina narxi t yildan so‘ng $V(t)=25000-3000t$ yevro qonuniyat bilan o‘zgaradi.

a) $V(0)$ qiymatni toping. Bu qiymat ma’nosini tushuntiring;
b) $V(3)$ qiymatni toping. Bu qiymat ma’nosini tushuntiring;
c) $V(t)=10000$ qiymatga necha yildan so‘ng erishiladi?

93. AQShda elektr montajchi chaqirilgani uchun \$60 va har bir soat uchun \$45 xizmat haqqini oladi.

a) $t=0, 1, 2, 3, 4, 5$ bo‘lganda mos jadvalni tuzing. C xizmat haqqining t vaqtga qanday bog‘liqligini grafik ko‘rinishda ifodalang.
b) $C(t)$ funksiyaning formulasini (algebraik ko‘rinishini) yozing.
c) $6\frac{1}{2}$ soat vaqt uchun qancha mablag‘ to‘lanadi?

94. Sisterna 265 l suv bilan to‘ldirilgan. Undan har bir minutda 11 l suv olinmoqda.

a) $t=0, 1, 2, 3, 4, 5$ bo‘lganda oqib chiqayotgan suvning V l hajmi t (minut) vaqtga qanday bog‘liqligini ifodalovchi jadval tuzing.
b) $V(t)$ bog‘lanishni grafik ko‘rinishda ifodalang.
c) $V(t)$ funksiyaning formulasini (algebraik ko‘rinishini) yozing.
d) 15 minutdan keyin sisternada qancha suv qoladi?
e) Sisterna qancha vaqtdan keyin bo‘shaydi?

95. Quyidagilardan qaysi biri kvadrat funksiya bo‘ladi:

a) $y=2x^2-4x+10$; c) $y=-2x^2$; e) $3y+2x^2-7=0$;
b) $y=15x-8$; d) $y=\frac{1}{3}x^2+6$; f) $y=15x^3+2x-16$?

96. (x, y) juftlik ko‘rsatilgan $y=ax^2+bx+c$ kvadrat funksiya bilan ifodalangan munosabatda bo‘ladimi:

a) $f(x)=6x^2-10$, $(0, 4)$;
b) $y=2x^2-5x-3$, $(4, 9)$;
c) $y=-4x^2+6x$, $(-\frac{1}{2}, -4)$;
d) $y=-7x^2+9x+11$, $(-1, -6)$;
e) $f(x)=3x^2-11x+20$, $(2, -10)$;
f) $f(x)=-3x^2+x+6$, $(\frac{1}{3}, 4)$?

97. $y=ax^2+bx+c$ kvadrat funksiya uchun y ning berilgan qiymatiga mos bo‘lgan x ning qiymatini toping:

a) $y=x^2+6x+10$, $y=1$; c) $y=x^2-5x+1$, $y=-3$;
b) $y=x^2+5x+8$, $y=2$; d) $y=3x^2$, $y=-3$.

98. Moddiy jism 80 m/s tezlikda balandlikka otilgan. Uning t sekundda yerga nisbatan balandligi $h(t)=80t-5t^2$ funksiya yordamida aniqlanadi.
a) 1 sekund, 3 sekund, 4 sekunddan keyin jismning balandligini toping;
b) qaysi vaqtda jismning balandligi: 140 m; 0 metr bo‘ladi? Javoblarga mos holatlarni tushuntiring.

99. Mahsulot ishlab chiqaruvchi tadbirkorning daromadi quyidagi formula bilan hisoblanadi:

$P(x)=-\frac{1}{2}x^2+36x-40$ (ming so‘m), bu yerda x – mahsulotlarning soni.

a) 0 ta mahsulot, 20 ta mahsulot ishlab chiqarilganda tadbirkor qanday daromadga ega bo‘ladi? b) 270 ming so‘m daromad olish uchun tadbirkor nechta mahsulot ishlab chiqishi kerak?

100. Funksiyalarning $x=-3, -2, -1, 0, 1, 2, 3$ qiymatlarga mos qiymatlarini toping. Natijalarni jadval ko‘rinishida bering va grafiklarni yasang:
a) $y=x^2+2x-2$; d) $f(x)=-x^2+x+2$; g) $y=x^2-5x+6$;
b) $y=x^2-3$; e) $y=x^2-4x+4$; h) $y=x^2+x+1$;
c) $y=x^2-2x$; f) $f(x)=-2x^2+3x+10$; i) $y=-x^2+x-1$.
Bu grafiklar qanday shaklda bo‘ladi?

101. Parabolalarning ordinatalar o‘qi bilan kesishish nuqtasini toping:
a) $y=x^2+2x+3$; d) $f(x)=3x^2-10x+1$; g) $y=8-x-2x^2$;
b) $y=2x^2+5x-1$; e) $y=3x^2+5$; h) $f(x)=2x^2-x^2-5$;
c) $y=-x^2-3x-4$; f) $y=4x^2-x$; i) $y=6x^2+2-5x$.

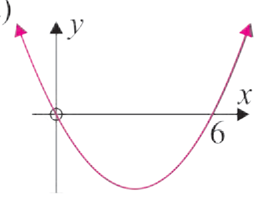
102. Funksiyalar grafiklari ordinatalar o‘qi bilan qanday nuqtalarda kesishadi:
a) $y=(x+1)(x+3)$; d) $y=(2x+5)(3-x)$; g) $y=(x-1)(x-6)$;
b) $y=(x-2)(x+3)$; e) $y=x(x-4)$; h) $y=-(x+2)(x+4)$;
c) $y=(x-7)^2$; f) $y=-(x+4)(x-5)$; i) $y=-(x-3)(x-4)$?

103. Parabolalarning abssissalar o‘qi bilan kesishish nuqtalarini toping:
a) $y=x^2-x-6$; d) $y=3x-x^2$; g) $y=-x^2-4x+21$; j) $y=-2x^2+x-5$;
b) $y=x^2-16$; e) $y=x^2-12x+36$; h) $y=2x^2-20x+50$; k) $y=-6x^2+x+5$;
c) $y=x^2+5$; f) $y=x^2+x-7$; i) $y=2x^2-7x-15$; l) $y=3x^2+x-1$.

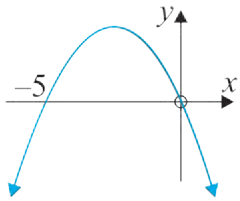
104. Parabolalarning koordinatalar o‘qlari bilan kesishish nuqtalarini toping:
a) $y=x^2+x-2$; d) $y=x^2+x+4$; g) $y=-x^2-7x$; j) $y=-x^2+2x-9$;
b) $y=(x+3)^2$; e) $y=3x^2-3x-36$; h) $y=-2x^2+3x+7$; k) $y=4x^2-4x-3$;
c) $y=(x+5)(x-2)$; f) $y=-x^2-8x-16$; i) $y=2x^2-18$; l) $y=6x^2-11x-10$.

105. Parabolaning simmetriya o‘qini toping:

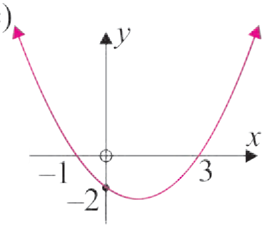
a)



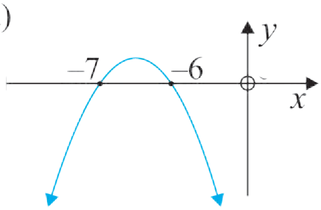
b)



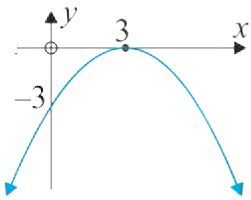
c)



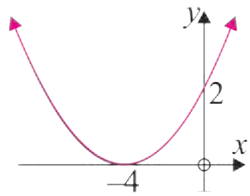
d)



e)



f)



106. Parabolaning simmetriya o‘qini toping:

a) $y=(x-2)(x-6)$;
b) $y=x(x+4)$;
c) $y=-(x+3)(x-5)$;
d) $y=(x-3)(x-8)$;
e) $y=2(x-5)^2$;
f) $y=3(x+2)^2$.

107. Parabolaning simmetriya o‘qini toping:

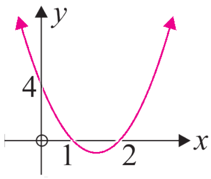
a) $y=x^2+6x+2$;
b) $y=x^2-8x-1$;
c) $f(x)=2x^2+5x-3$;
d) $y=-x^2+3x-7$;
e) $y=2x^2-5$;
f) $y=-5x^2+7x$;
g) $f(x)=x^2-6x+9$;
h) $y=10x-3x^2$;
i) $y=\frac{1}{8}x^2+x-1$.

108. Parabola uchining koordinatalarini toping:

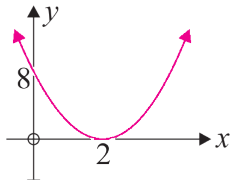
a) $y=x^2-4x+7$;
b) $y=x^2+2x+5$;
c) $f(x)=-x^2+6x-1$;
d) $y=x^2+3$;
e) $f(x)=2x^2+12x$;
f) $y=-3x^2+6x-4$;
g) $y=x^2-x-1$;
h) $y=-2x^2+3x-2$;
i) $y=-\frac{1}{4}x^2+3x-2$.

109. Parabolaga qarab, unga mos kvadrat funksiya formulasini yozing:

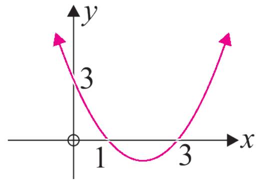
a)

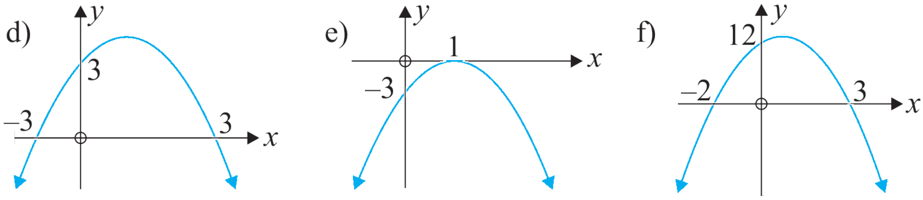


b)

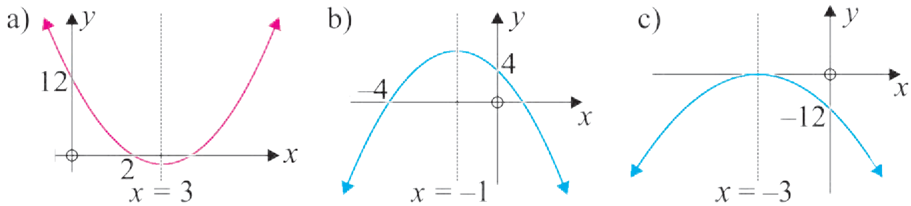


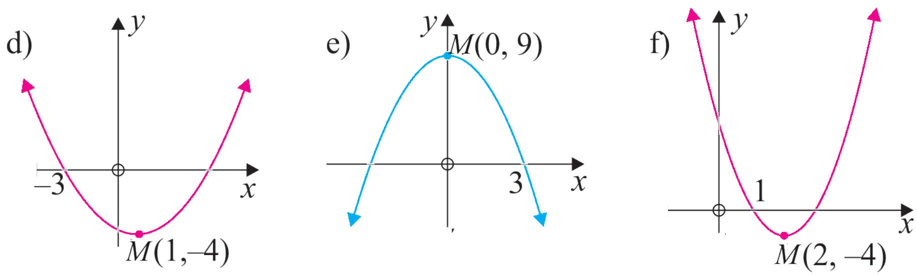
c)





110. Parabolaga qarab, kvadrat funksiya formulasini yozing:





111. Dilshod dengizga durni olish uchun sho'ng'idi. Uning t sekunddan keyingi sho'ng'ish chuqurligi $H(t)=-4t^2+4t+3$ metr bo'ldi, $t \geqslant 0$.

a) durlar qanday chuqurlikda joylashgan?

b) Dilshod durni olish uchun qancha vaqt sarflaydi?

c) Dilshod qanday balandlikdan suvga sho'ng'idi?

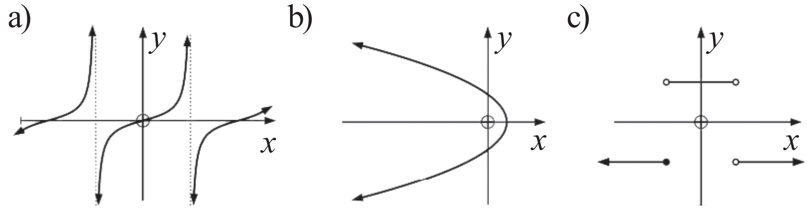
112. Jasmina ko'ylak tikish uchun buyurtma oldi. U bir kunda x ta ko'ylak tiksa, $P(x)=-x^2+20x$ AQSh dollari miqdorida daromad oladi.

a) Eng katta daromad olish uchun u qancha ko'ylak tikish kerak?

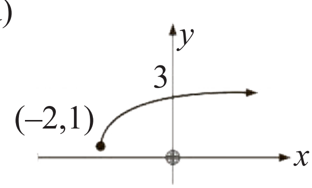
b) Eng katta daromad necha dollarga teng?

Nazorat ishi namunasi

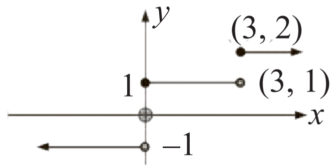
1. Quyidagi munosabatlardan qaysilari funksiyalardir?



d)



e)

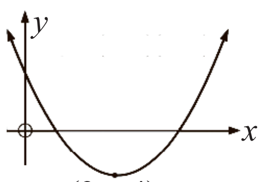


2. Quyidagi tartiblangan juftliklar to‘plamlaridan qaysilari akslantirish bo‘ladi? Javobingizni asoslang.

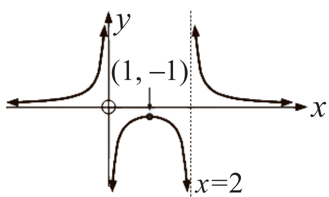
a) $\{(1, 2),(-1, 2),(0, 5),(2, -7)\}$; b) $\{(0, 1),(1, 3),(2, 5),(0, 7)\}$;
c) $\{(6, 1),(6, 2),(6, 3),(6, 4)\}$.

3. Grafik ko‘rinishda berilgan funksiyalarning aniqlanish sohasini va qiymatlar to‘plamini toping:

a)

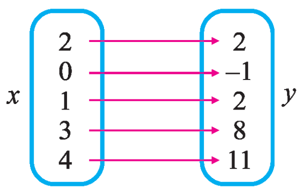


b)

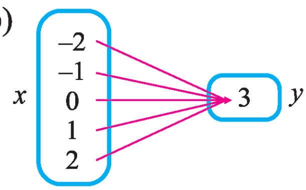


4. Quyidagi diagramma $y=f(x)$ akslantirishni bermoqda:

a)



b)



1) $y=f(x)$ akslantirishning aniqlanish sohasini va qiymatlar to‘plamini yozing.

2) $y=f(x)$ akslantirish tekislikdagi koordinatalar sistemasida qanday tasvirlanadi?

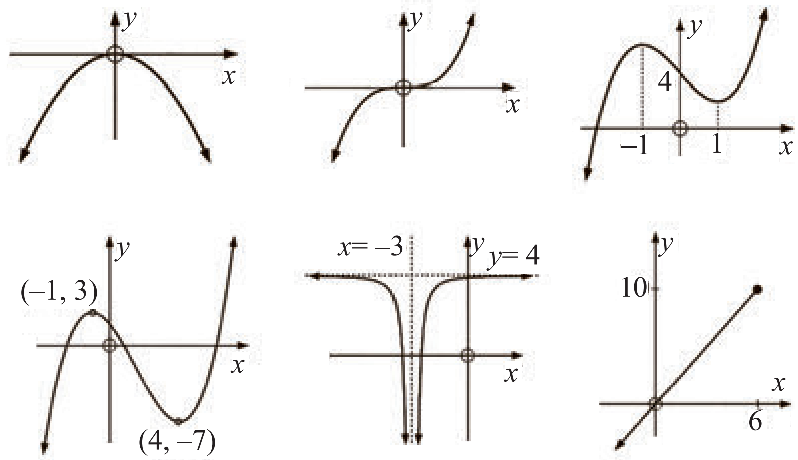
3) $y=f(x)$ uchun aniq ifodani yozing.

5. $f(x)=2x-x^2$ funksiya uchun:

a) $f(2)$; b) $f(-3)$; c) $f\left(-\frac{1}{2}\right)$ qiymatlarni toping.

6. $g(x)=x^2-3x$ funksiya uchun a) $g(x+1)$; b) $g(x^2-2)$ ifodalarni toping va soddalashtiring.

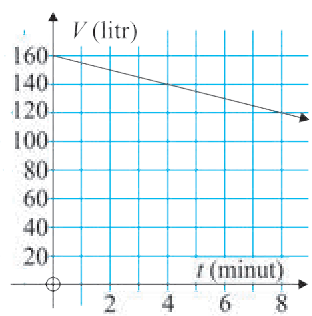
7. Grafik ko‘rinishda berilgan funksiyalarning kamayish va o‘sish oraliqlarini toping:



8. a) $f(x)=2x+1$; b) $f(x)=-3x+2$; c) $f(x)=x^2$; d) $f(x)=-x^3$ funksiyalar uchun:

1) funksiyalarning o‘qlar bilan kesishish nuqtalarini toping;
2) lokal maksimum, lokal minimum nuqtalari koordinatalarini toping;
3) funksiyalar grafigini taxminiy chizing.

9. Quyidagi grafikda minutlarda ifodalangan t vaqtda sisternadan sizib chiqayotgan neft mahsulotining V hajmi tasvirlangan.

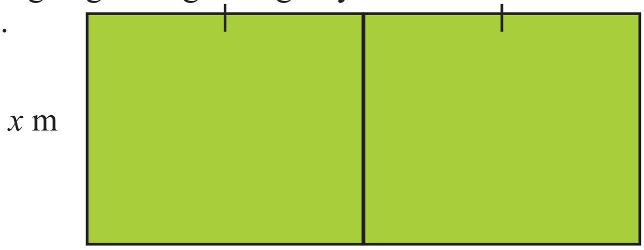


1) Sizib chiqayotgan neft maxsulotining hajmi bilan vaqt orasidagi bog‘lanish formulasini toping.
2) 15 minutda qancha neft sizib chiqadi?
3) 50 litr neft necha minutda sizib chiqadi?
4) Sisterna qancha vaqtdan so‘ng bo‘shaydi?

10. Tosh dengiz sathidan 60 metr balandlikdan yuqoriga uloqtirilgan. t sekunddan so‘ng toshning dengiz sathiga nisbatan balandligi $H(t)=-5t^2+20t+60$ metrga teng bo‘lsa:

1) Necha sekunddan so‘ng toshning balandligi eng katta bo‘ladi?
2) Toshning dengiz sathiga nisbatan balandligi qanchaga teng?
3) Necha sekunddan so‘ng tosh suvga tushadi?

11. Fermer rasmda ko‘rsatilgan ikkita yonma-yon turgan bir xil maydonga ega bo‘lgan bug‘doy dalasini 2000 metr devor bilan o‘radi.



1) Dalalarning umumiy maydoni x orqali qanday ifodalanadi?
2) Ikkita dalaning umumiy maydoni eng ko‘pi bilan nehca kvadtar metrga teng bo‘lishi mumkin? Bu dalalarning o‘lchamlarini aniqlang.

55 DAVRIY JARAYONLAR VA ULARNI KUZATISH

Davriy jarayonlar tabiatda va texnikada keng tarqalgan. Ularga misollar keltiraylik:

- yil fasllari bo‘yicha ob-havoning o‘zgarishi;
- oylardagi o‘rtacha temperaturaning o‘zgarishi;
- kun va tunning davomiyligi;
- dengiz qirg‘og‘i yonidagi suv chuqurligi;
- hayvonlar soni;
- quyosh faolligining o‘zgarishi;
- mexanika, elektrotexnikadagi davriy tebranishlar.

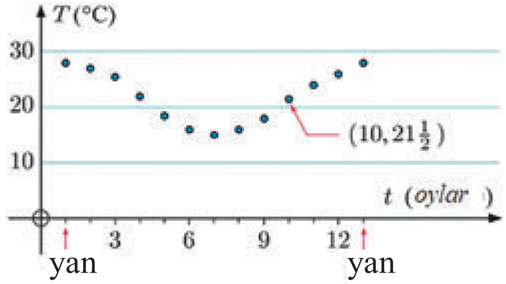
Bu jarayonlarda muayyan vaqt oraliqlarida takrorlanib turadigan holatlar kuzatiladi. Ular vaziyatga qarab davriy, tebranadigan yoki siklik deyiladi.

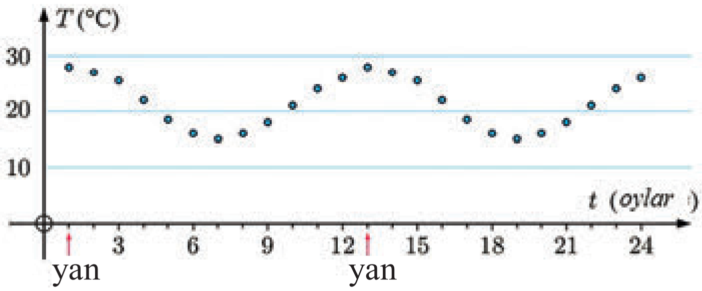
Masalan, Janubiy Afrikadagi Keyptaun shahrida oylik maksimal temperaturaning o‘zgarishini ifodalovchi jadvalni qaraylik:

Oy	Yan	Fev	Mar	Apr	May	Iyun	Iyul	Avg	Sen	Okt	Noy	Dek
Temp (0 °C)	28	27	$25\frac{1}{2}$	22	$18\frac{1}{2}$	16	15	16	18	$21\frac{1}{2}$	24	26

Bu ma’lumotlarni grafik ko‘rinishda ifodalaylik. Buning uchun ordinatalar o‘qi temperaturani, abssissalar o‘qi esa oyning tartib raqamini (masalan, fevral uchun $t=2$) bildirsin. Grafikda yanvar oyida o‘rtacha 28 °C temperatura bo‘lishi kuzatilmoqda. Bunday qiymatning har yilning yanvarida, ya’ni har 12 oyda takrorlanishini kutish tabiiy.

Boshqa oylar uchun ham o'rtacha temperatura o'zgarishini taqribiy aks ettiruvchi grafikni chizib, uni keyingi yilga ham davom ettirsak bo'ladi:





Agar $y=f(t)$ funksiya t oyda o'rtacha temperaturani ifodalasa, $f(0)=f(12)=f(24)=...$, $f(1)=f(13)=f(25)=...$ va h.k. kabi qonuniyat, umumiy holda, ixtiyoriy t uchun $f\ (t+12)$ bo'lishi kuzatilmoqda.

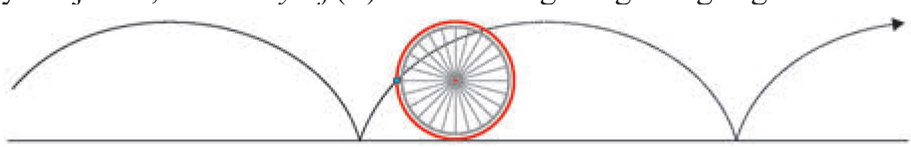
Bunda takrorlanish kuzatiladigan 12 oy muddatni davr deb aytamiz.

X to'plamda aniqlangan $f(x)$ funksiya uchun ixtiyoriy x da $f(x+T)=f(x)$ tenglikni qanoatlantiradigan $T>0$ mavjud bo'lsa, $f(x)$ funksiya *davriy* deyiladi, bunda $x+T \in X$.

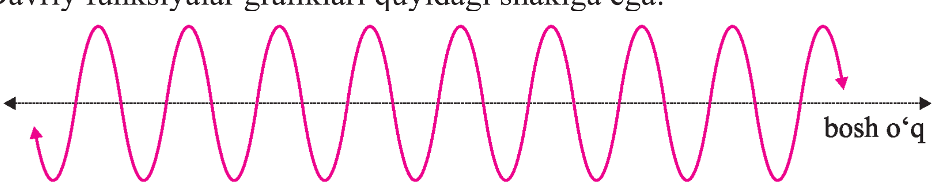
Ravshanki, $f(x+T)=f(x)$ bo'lsa, u holda $f(x)=f(x+T)=f(x+2T)=...$ Bunday $T>0$ sonlarning eng kichik qiymatini ***funksiyaning davri*** deb ataymiz.

G'ildirak to'g'ri chiziq bo'ylab aylanib harakat qilsa, undagi tayin bir belgilangan nuqta *sikloida* deb nomlangan egri chiziq bo'yicha davriy harakat qiladi.

Aytish joizki, sikloida $y=f(x)$ ko'rinishdagi tenglamaga ega emas.

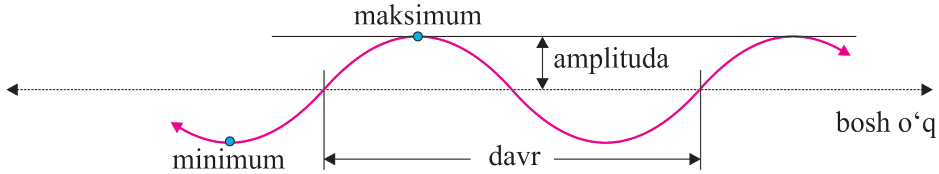


Davriy funksiyalar grafiklari quyidagi shaklga ega:



Bosh o‘q tenglamasi quyidagicha topiladi: $y = \dfrac{\max + \min}{2}$, bunda max – funksiyaning eng katta, min esa eng kichik qiymati.

Davriy funksiya grafigi quyidagi tarkibiy qismlarga ega:



Amplituda funksiyaning maksimumi bilan o‘q (yoki o‘q bilan minimum) orasidagi masofa bo‘lib, u quyidagicha topiladi:

$$\text{amplituda} = \frac{\max - \min}{2}.$$

Savol va topshiriqlar

1. Davriy jarayonga misol keltiring.
2. Funksiyaning davriga ta’rif bering.
3. Davriy funksiyaning amplitudasi qanday hisoblanadi?
4. Sikloidaning nimaligini tushuntiring.
5. Qachon kvadrat funksiya maksimumga (minimumga) ega?

Mashqlar

113. Har bir hol uchun ma’lumotlarni grafik ko‘rinishda tasvirlang va ularning davriy-davriymasligi to‘g‘risida xulosa chiqaring:

a)	x	0	1	2	3	4	5	6	7	8	9	10	11	12
	y	0	1	1,4	1	0	–1	–1,4	–1	0	1	1,4	1	0

b)	x	0	1	2	3	4
	y	4	1	0	1	4

c)	x	0	0,5	1,0	1,5	2,0	2,5	3,0	3,5
	y	0	1,9	3,5	4,5	4,7	4,3	3,4	2,4

d)	x	0	2	3	4	5	6	7	8	9	10	12
	y	0	4,7	3,4	1,7	2,1	5,2	8,9	10,9	10,2	8,4	10,4

114. Quyidagi jadvalda g‘ildirak to‘g‘ri chiziq bo‘ylab aylanib harakat qilsa, unda belgilangan nuqtaning harakatini ifodalovchi kattaliklar keltirilgan:

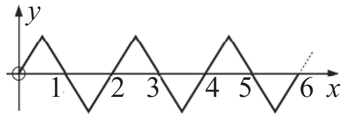
Masofa (sm)	0	20	40	60	80	100	120	140	160	180	200
Balandlik (sm)	0	6	23	42	57	64	59	43	23	7	1

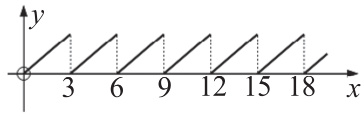
Masofa (sm)	220	240	260	280	300	320	340	360	380	400
Balandlik (sm)	5	27	40	55	63	60	44	24	9	3

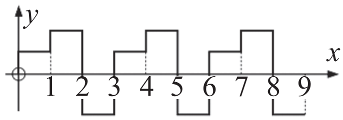
a) Balandlikning masofaga bog‘liqligini grafik ko‘rinishda ifodalang.

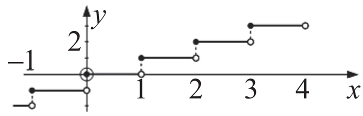
b) Bu jarayon davriymi? Agar davriy bo‘lsa, o‘q tenglamasini, funksiyaning maksimumini, davrini, amplitudasini toping.

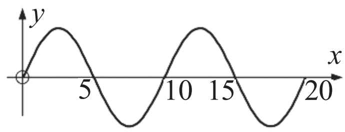
115. Grafiklardan qaysi biri davriy jarayonni ifodalaydi?

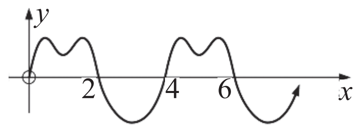
a) 

b) 

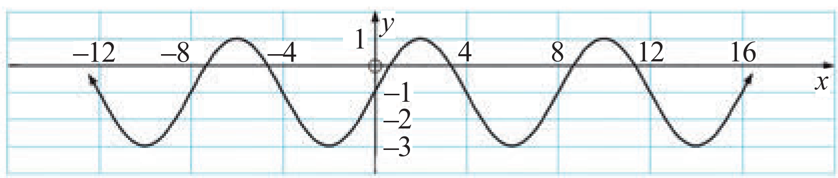
c) 

d) 

e) 

f) 

116.



Berilgan davriy funksiya uchun:

a) amplitudani toping; b) davrni toping;

c) birinchi maksimum nuqtasini toping;

d) ikkita maksimum nuqta orasidagi masofani aniqlang;

e) bosh o‘qning tenglamasini tuzing.

56-58 y=sin$x$, y=cos$x$ FUNKSIYALAR VA ULAR YORDAMIDA MODELLASHTIRISH

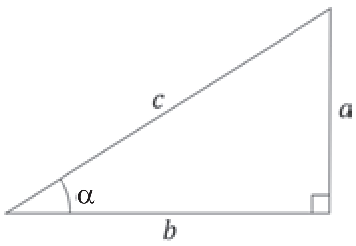
To‘g‘ri burchakli uchburchakda *a, b* – katetlar, *c* – gipotenuza bo‘lsin. α deb *a* katetga qarama-qarshi burchakni belgilaymiz (1- rasmga qarang).

Geometriya kursida α burchakning sinusi va kosinusi quyidagi tengliklar yordamida kiritiladi:

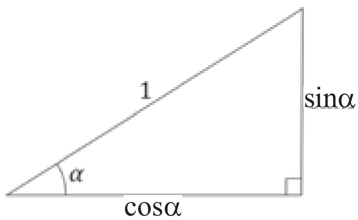
$$\sin\alpha = \frac{a}{c}, \quad \cos\alpha = \frac{b}{c}.$$

Gipotenuzani 1 deb olsak, 1- rasm 2- rasmdagi ko‘rinishni oladi.

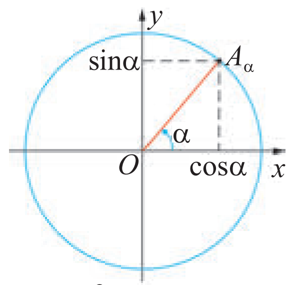
Tekislikda koordinatalar sistemasini kiritib, unda *radiusi 1 ga teng aylanani– birlik aylanani* qaraymiz va shu aylanada α burchakka mos bo‘lgan nuqtani belgilaymiz (3- rasm).



1-rasm.



2-rasm.

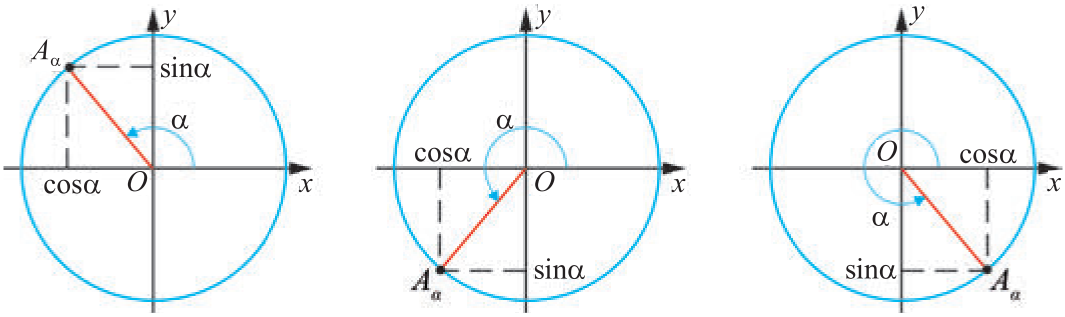


3-rasm.

α burchakning sinusi deb (1;0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo‘lgan A_α nuqtaning ordinatasiga aytiladi (sinα kabi belgilanadi).

Huddi shunday, α burchakning kosinusi deb (1; 0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo‘lgan A_α nuqtaning abssissasiga aytiladi (cosα kabi belgilanadi).

α burchakka mos nuqta boshqa choraklarda yotsa, quyidagi kabi shakllarga ega bo‘lamiz (4- rasm):



4-rasm.

Pifagor teoremasiga ko‘ra, $\cos^2\alpha + \sin^2\alpha = 1$ – *asosiy trigonometrik ayniyat* o‘rinli, bunda $0^\circ \le \alpha \le 360^\circ$. Trigonometriyada qaraladigan burchak (yoy) lar graduslarda yoki radianlarda o‘lchanishi mumkin.

α markaziy burchakka mos yoy uzunligining o‘sha yoy radiusiga nisbati shu burchakning radian o‘lchovi deyiladi.

Graduslarda berilgan α burchakning radian o‘lchovi $\frac{\pi}{180^\circ}\alpha$ ga teng.

Ko‘p uchraydigan burchaklarning radian o‘lchovlari jadvalini keltiramiz:

Gradus	0°	30°	45°	60°	90°	180°	270°	360°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

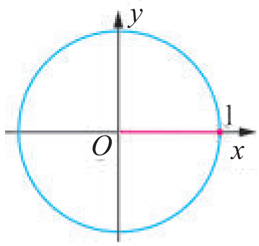
Ayrim α burchaklar sinusi va kosinusi qiymatlarini topaylik.

1. $\alpha = 0°$ bo‘lsin (5- rasm). Bu holga mos nuqtaning abssissasi 1 ga, ordinatasi esa 0 ga teng, demak, $\sin 0° = 0$, $\cos 0° = 1$.

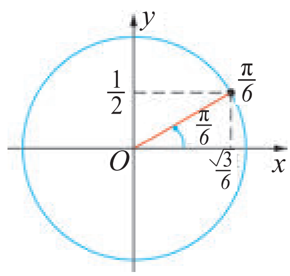
2. $\alpha = \pi/6 = 30°$ bo‘lsin (6- rasm). To‘g‘ri burchakli uchburchakda 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng bo‘lgani bois, $\sin\frac{\pi}{6} = \frac{1}{2}$ bo‘ladi. Asosiy trigonometrik ayniyatga ko‘ra $\cos\frac{\pi}{6} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$.

3. $\alpha = \pi/4 = 45°$ bo‘lsin (7- rasm). Bu holda teng yonli to‘g‘ri burchakli uchburchak hosil bo‘ladi.

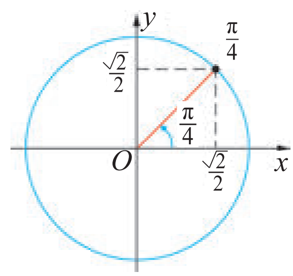
Bunday uchburchakda α burchakning sinusi va kosinusi o‘zaro tengdir. Ularni x deylik. Asosiy trigonometrik ayniyatdan $x^2 + x^2 = 1$, ya’ni $x = \frac{\sqrt{2}}{2}$ bo‘ladi. Demak, $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$



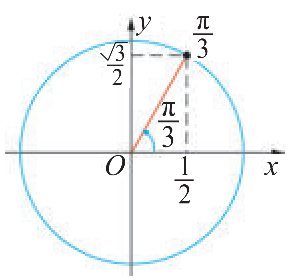
5- rasm.



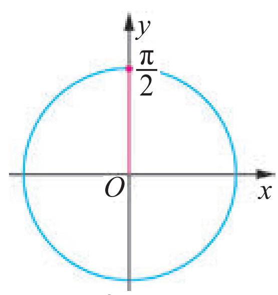
6- rasm.



7- rasm.



8- rasm.

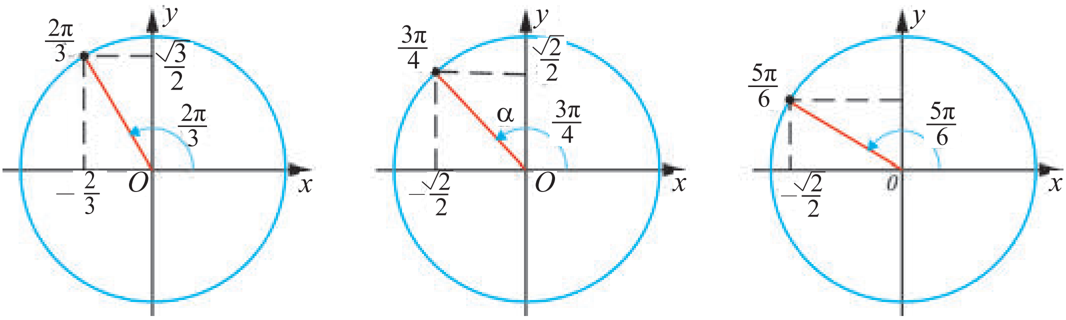


9- rasm.

4. $\alpha = \pi/3 = 60°$ bo‘lsin (8- rasm). Bu holda xuddi $\alpha = \frac{\pi}{6}$ holga o‘xshash mulohaza yuritib, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ tengliklarga ega bo‘lamiz.

5. $\alpha = \pi/2 = 90°$ bo‘lsin (9- rasm). Bu holga mos nuqtaning abssissasi 0 ga,

ordinatasi esa 1 ga teng. Demak, $\cos\frac{\pi}{2}=0$, $\sin\frac{\pi}{2}=1$.



10- rasm.

6. $\frac{2\pi}{3}=120°$, $\frac{3\pi}{4}=135°$, $\frac{5\pi}{6}=150°$ bo‘lgan hollarni qaraylik (10- rasm). $\frac{2\pi}{3}$ nuqta uchun $\frac{2\pi}{3}=\pi-\frac{\pi}{3}$. U holda bu nuqta $\frac{\pi}{3}$ nuqtaga *Oy* o‘qiga nisbatan simmetrik. Demak, $\cos\frac{2\pi}{3}=-\frac{1}{2}$, $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$.

$\frac{3\pi}{4}$ nuqta uchun $\frac{3\pi}{4}=\pi-\frac{\pi}{4}$. U holda bu nuqta $\frac{\pi}{4}$ nuqtaga *Oy* o‘qiga nisbatan simmetrik. Demak, $\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$, $\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$.

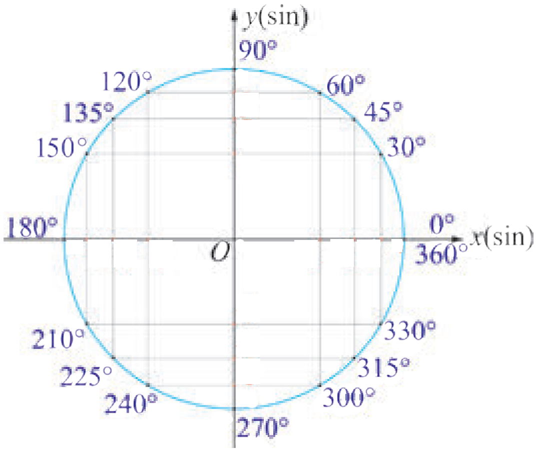
$\frac{5\pi}{6}$ nuqta uchun $\frac{5\pi}{6}=\pi-\frac{\pi}{6}$. U holda bu nuqta $\frac{\pi}{6}$ nuqtaga *Oy* o‘qiga nisbatan simmetrik. Demak, $\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$, $\sin\frac{5\pi}{6}=\frac{1}{2}$.

7. $\alpha=\pi=180°$ holda $\cos\pi=-1$, $\sin\pi=0$ ekanini isbotlash va mos rasm chizishni o‘quvchiga havola qilamiz.

Yuqorida biz $[0; \pi]$ oraliqda ayrim burchaklar uchun sinus va kosinus qiymatlarini aniqladik. Bu burchaklarning har biriga π ni qo‘shib, $[\pi; 2\pi]$ oraliqdagi burchaklar uchun ham sinus va kosinus qiymatlarini aniqlash mumkin.

Natijalarni *trigonometrik aylana* deb nomlangan 11- rasmda ifodalaymiz:

Yuqoridagi qiymatlardan foydalanib, $y=\sin x$, $y=\cos x$ funksiyalar grafiklarini yasash mumkin. Buning uchun abssissalar o‘qida α burchakning qiymatlarini, ordinatalar o‘qida esa sinusning mos qiymatlarini olib, hosil bo‘lgan nuqtalarni belgilaymiz. So‘ng belgilangan nuqtalarni silliq chiziq bilan tutashtirib, $[0; 2\pi]$ oraliqdagi $y=\sin x$ (12- rasm) funksiya grafigini hosil qilamiz. $y=\cos x$ (13- rasm) grafigi ham shu kabi yasaladi.



$$30^\circ = \frac{\pi}{6};\quad 45^\circ = \frac{\pi}{4};\quad 60^\circ = \frac{\pi}{6};$$

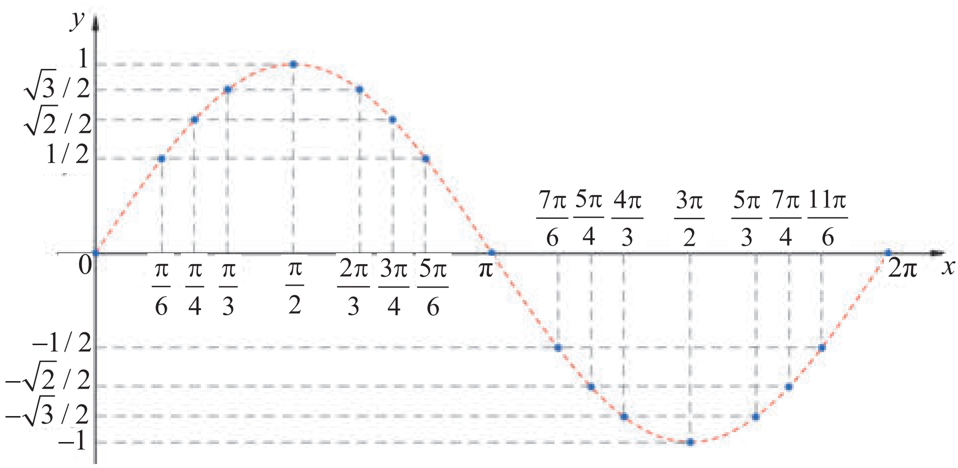
$$90^\circ = \frac{\pi}{2};\quad 120^\circ = \frac{2\pi}{3};\quad 135^\circ = \frac{3\pi}{6};$$

$$180^\circ = \pi;\quad 210^\circ = \frac{7\pi}{6};\quad 225^\circ = \frac{5\pi}{4};$$

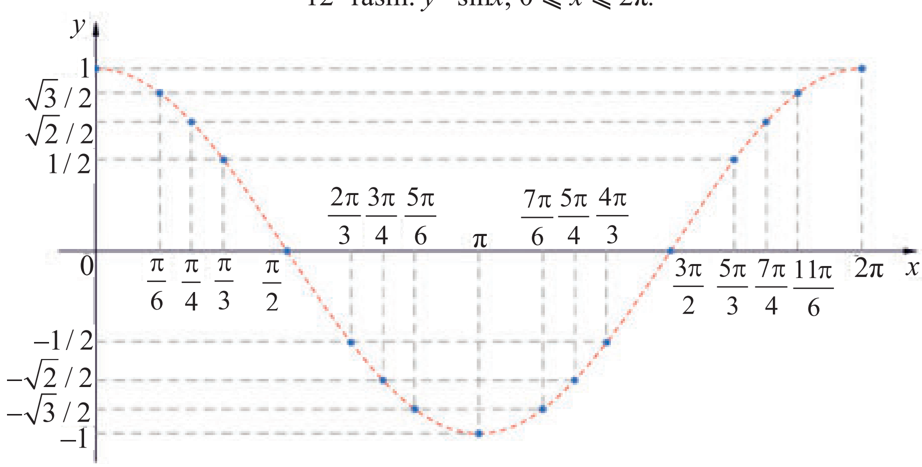
$$240^\circ = \frac{4\pi}{3};\quad 270^\circ = \frac{3\pi}{2};\quad 300^\circ = \frac{5\pi}{3};$$

$$315^\circ = \frac{7\pi}{4};\quad 330^\circ = \frac{11\pi}{6}.$$

11- rasm. Trigonometrik aylana. Sinus va kosinusning ayrim qiymatlari.

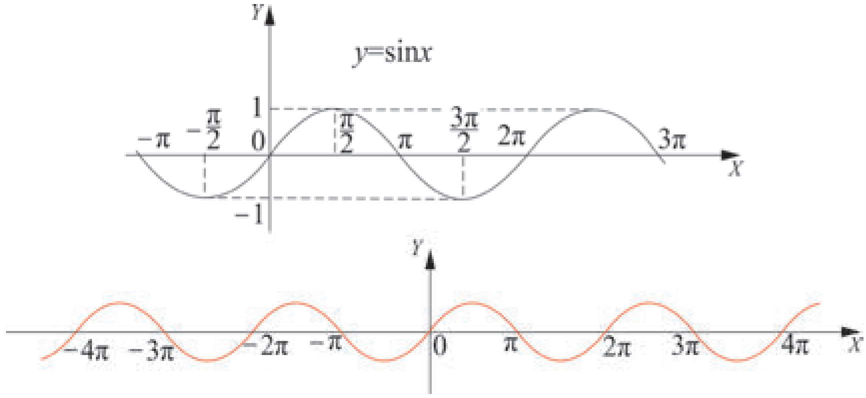


12- rasm. $y=\sin x,\ 0 \leqslant x \leqslant 2\pi$.

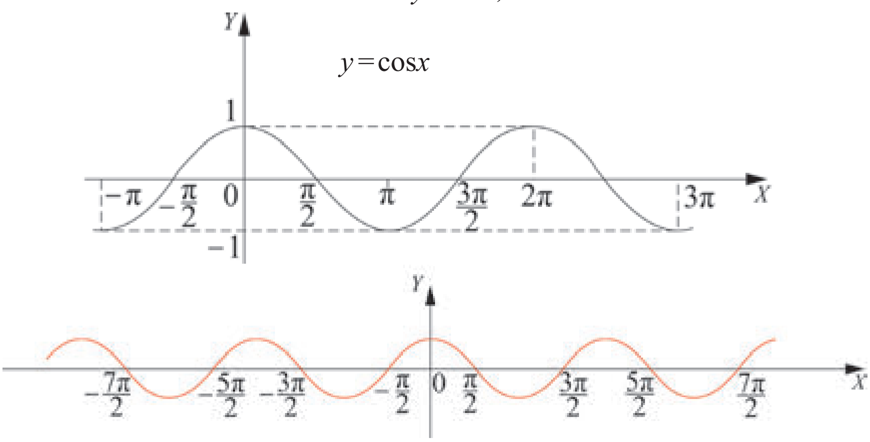


13- rasm. $y=\cos x,\ 0 \leqslant x \leqslant 2\pi$.

Bu grafiklarni davriy ravishda davom ettirib, $y=\sin x$, $y=\cos x$ funksiyalarning grafiklarini hosil qilamiz (14 va 15- rasmlar).



14- rasm. $y=\sin x$, $x \in R$.



15- rasm. $y=\cos x$, $x \in R$.

Grafiklarni o‘qib, shunday xulosaga kelamiz: $y=\sin x$ ($y=\cos x$) funksiyaning davri 2π ga, amplitudasi 1 ga, eng katta qiymati 1 ga, eng kichik qiymati esa -1 ga teng.

Tatbiqlarda keng uchraydigan $y=a\sin x$ va $y=\sin bx$, $b \neq 0$ funksiyalar to‘g‘risida ba’zi mulohazalarni keltiramiz.

$y=a\sin x$ funksiyaning amplitudasi $|a|$ ga teng. Uning grafigi $y=\sin x$ funksiya grafigini $|a|>1$ bo‘lganda ordinata o‘qi bo‘yicha cho‘zish, $|a|<1$ bo‘lganda esa siqish natijasida hosil bo‘ladi. $y=\sin bx$ funksiyaning davri $\dfrac{360°}{|b|}$ ga teng. Bu funksiyaning grafigi $y=\sin x$ funksiya grafigidan $0<|b|<1$ bo‘lganda abssissa o‘qi bo‘yicha cho‘zish, $|b|>1$ bo‘lganda siqish natijasida hosil bo‘ladi.

$y=\sin x+c$ ko‘rinishdagi funksiya grafigi $y=\sin x$ funksiya grafigini c birlikka parallel ko‘chirish natijasida hosil bo‘ladi va bunda $y=\sin x+c$ funksiyaning bosh o‘qi $y=c$ tenglamaga ega.

Yuqoridagilarni inobatga olib, $y=a\sin bx+c$ ko‘rinishdagi funksiya grafigini hosil qilish mumkin.

Masalan, $y=2\sin 3x+1$ funksiyani qaraylik.

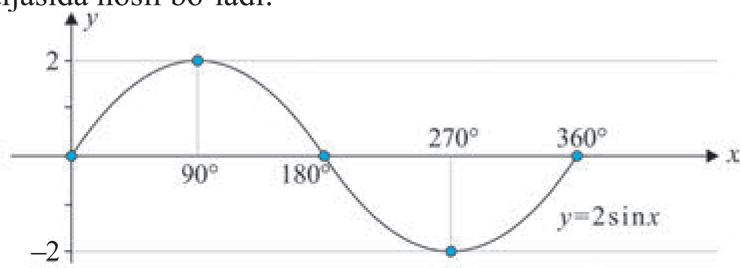
Bu funksiya grafigi $y=\sin x$ funksiya grafigidan quyidagicha hosil bo‘ladi:

1. Amplitudani ikkiga ko‘paytirib $y=2\sin x$ ni hosil qilamiz
2. Davrni uchga bo‘lib, $y=2\sin 3x$ ni hosil qilamiz
3. Berilgan 1 birlikka parallel ko‘chiramiz. $y=2\sin 3x+1$ funksiyaning bosh o‘qi $y=1$ tenglamaga ega.
4. Natijada $y=2\sin 3x+1$ funksiya grafigini hosil qilamiz.

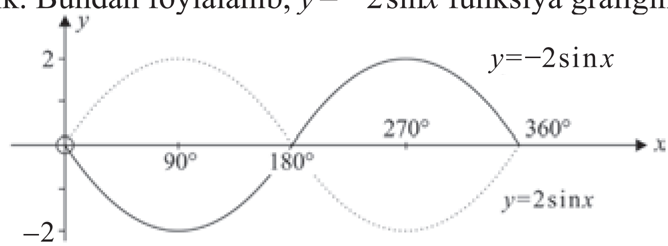
Shunga o‘xshash mulohazalarni $y=\cos x$ funksiya haqida ham keltirsa bo‘ladi.

1- misol. $y=2\sin x$, $y=-2\sin x$, $y=\sin 2x$ funksiyalar grafiklarini yasang, $0°\leq x\leq 360°$.

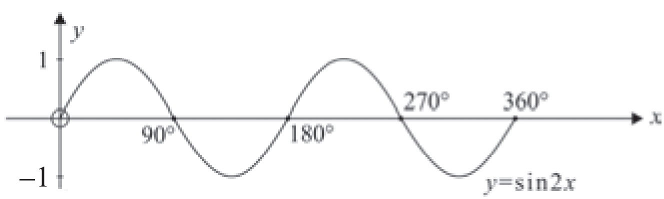
△ Dastlab $y=2\sin x$ funksiya grafigini yasaymiz. Bu funksiyaning amplitudasi 2 ga teng va uning grafigi $y=\sin x$ funksiya grafigini ordinatalar o‘qi bo‘yicha cho‘zish natijasida hosil bo‘ladi:



$y=-2\sin x$ funksiya grafigi $y=2\sin x$ funksiya grafigiga abssissa o‘qiga nisbatan simmetrik. Bundan foylalanib, $y=-2\sin x$ funksiya grafigini yasaymiz.

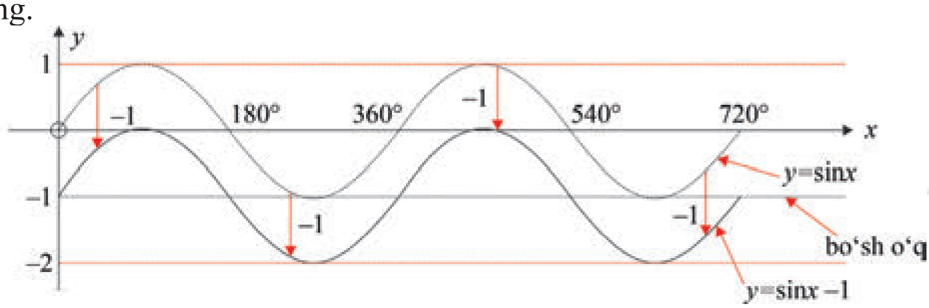


$y=\sin 2x$ funksiyaning davri $\frac{360°}{2}=180°$. Bu funksiya grafigi quyidagicha bo‘ladi:



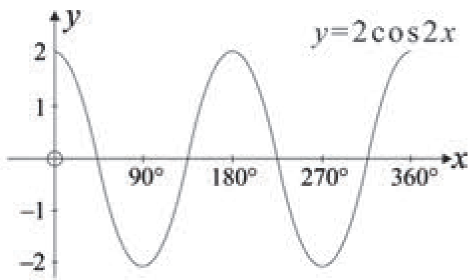
▲

2- misol. $0°\leqslant x\leqslant 720°$ bo'lganda $y=\sin x$ va $y=\sin x-1$ funksiyalar grafiklarini yasang.



3- misol. $0°\leqslant x\leqslant 360°$ kesmada $y=2\cos 2x$ funksiya grafigini yasaylik.

$a=2$. Demak, funksiya amplitudasi $|2|=2$ bo'ladi, $b=2$ bo'lgani uchun funksiyaning davri esa $\frac{360°}{|b|}=\frac{360°}{2}=180°$ bo'ladi. Bundan ushbu grafikka ega bo'lamiz:



Savol va topshiriqlar

1. Birlik doirada burchak sinusiga ta'rif bering.
2. Birlik doirada burchak kosinusiga ta'rif bering.
3. 30° li burchak uchun sinus va kosinusni hisoblang.
4. $y=\sin x$ funksiya grafigini chizing.
5. $y=\cos x$ funksiya grafigini chizing.

Mashqlar

117. Grafiklarni $0°\leqslant x\leqslant 360°$ kesmada yasang:

a) $y=3\sin x$; | b) $y=-3\sin x$; | c) $y=\frac{3}{2}\sin x$; | d) $y=-\frac{3}{2}\sin x$.

118. Grafiklarni $0°\leqslant x\leqslant 540°$ kesmada yasang:

a) $y=\sin 3x$; | b) $y=\sin(\frac{x}{2})$; | c) $y=\sin(-2x)$; | d) $y=-\sin\frac{x}{3}$.

119. Funksiyaning davrini aniqlang:

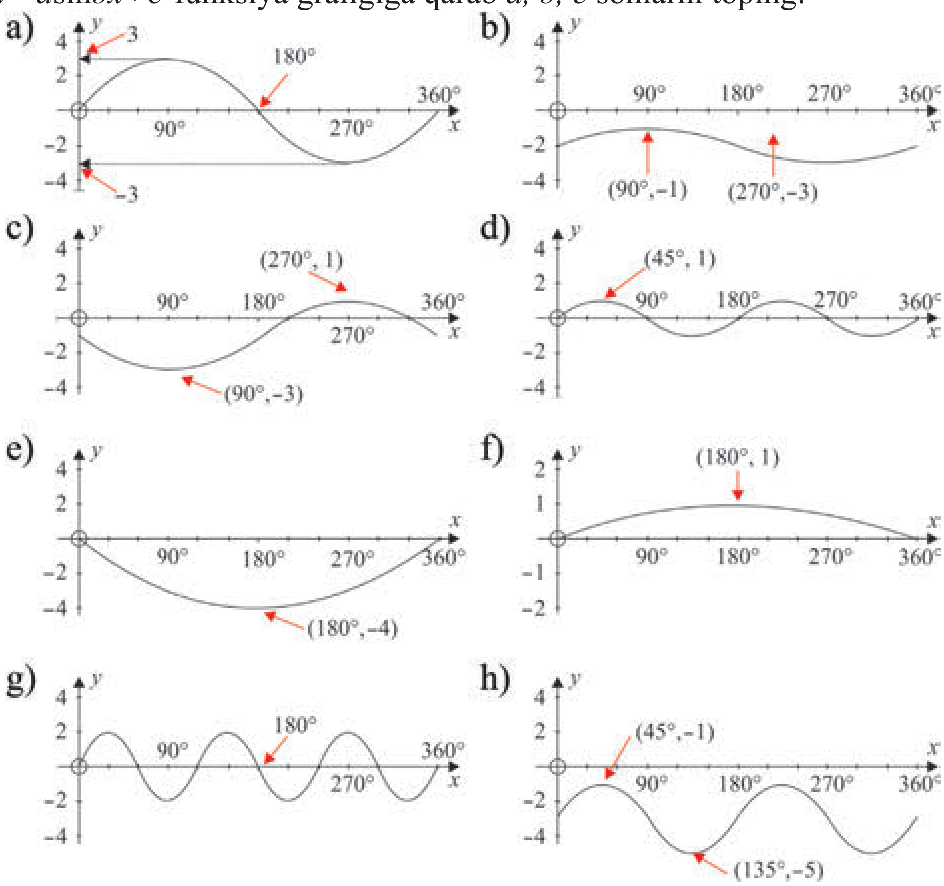
a) $y=\sin 4x$; | b) $y=\sin(-4x)$; | c) $y=\sin(\frac{x}{3})$; | d) $y=\sin(0{,}6x)$.

120. Agar $y=\sin bx$, $b>0$ uchun funksiyaning davri

a) 900°; | b) 120°; | c) 2160°; | d) 720°

ga teng bo'lsa, b ni toping.

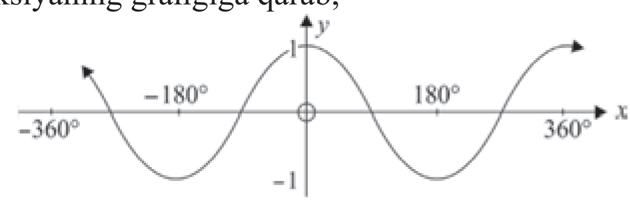
121. $y=a\sin bx+c$ funksiya grafigiga qarab a, b, c sonlarni toping:



122. Grafiklarni $0° \leqslant x \leqslant 360°$ kesmada yasang:

a) $y=\sin x+1$; b) $y=\sin x-2$; c) $y=1-\sin x$;
d) $y=2\sin x-1$; e) $y=\sin 3x+1$; f) $y=1-\sin 2x$.

123. $y=\cos x$ funksiyaning grafigiga qarab,



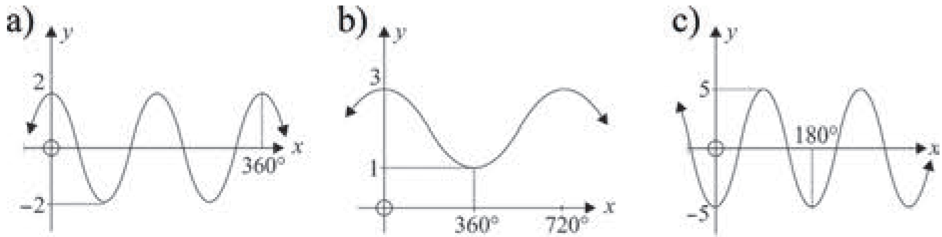
a) $y=\cos x+2$; b) $y=\cos x-1$; c) $y=\dfrac{2}{3}\cos x$;
d) $y=\dfrac{3}{2}\cos x$; e) $y=-\cos x$; f) $y=\cos 2x$;
g) $y=\cos(\dfrac{x}{2})$; h) $y=3\cos 2x$ funksiyalar grafiklarini yasang.

124 Funksiyaning davrini aniqlang:

a) $y=\cos 3x$; b) $y=\cos(\frac{x}{3})$; c) $y=\cos(\frac{x}{2})$; d) $y=\cos 4x$.

125. $y=a\cos bx+c$ funksiya berilgan bo‘lsin. *a, b, c* sonlarning geometrik ma’nosini aniqlang.

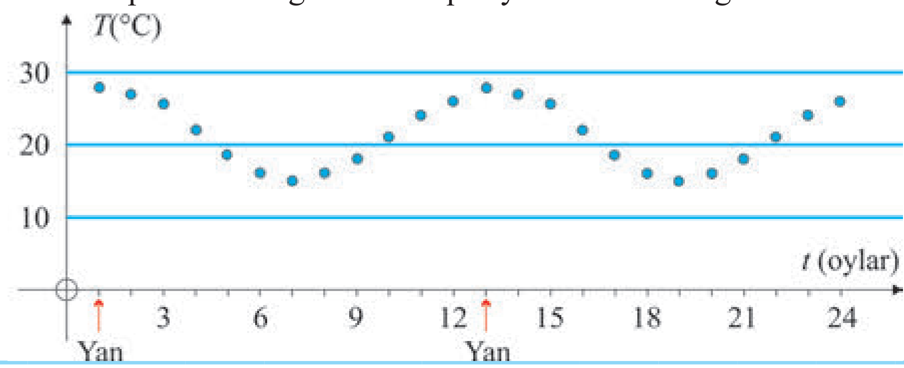
126. $y=a\cos bx+c$ funksiya grafigiga qarab *a, b, c* sonlarni toping.



4- misol. Quyida Janubiy Afrikadagi Keyptaun shahrida ob-havoning oylik maksimal temperaturasining o‘zgarishini ifodalovchi jadval berilgan:

Oy	*Yan*	*Fev*	*Mar*	*Apr*	*May*	*Iyun*	*Iyul*	*Avg*	*Sen*	*Okt*	*Noy*	*Dek*
T(°C)	28	27	$25\frac{1}{2}$	22	$18\frac{1}{2}$	16	15	16	18	$21\frac{1}{2}$	24	26

Maksimal temperatura o‘zgarishini taqribiy aks ettiruvchi grafikni keltiramiz:



Bu jarayonning modeli $T=a\cos bt+c$ ko‘rinishda bo‘lsin deb faraz qilib, parametrlar – *a, b, c* larni topamiz. Davr 12 oy bo‘lgani uchun

$$\frac{360°}{|b|}=12\text{, ya'ni } b=\frac{360°}{12}=30°.$$

Amplitudani hisoblaymiz: $\frac{\max-\min}{2}\approx\frac{28-15}{2}=6{,}5$. Bundan α≈6,5.

Bosh o‘q maksimal va minimal qiymatlar to‘g‘ri chiziqlari o‘rtasida bo‘lgani bois $c\approx\frac{28+15}{2}\approx 21{,}5$.

Demak, maksimal oylik temperatura vaqt o‘tishi bilan o‘zgarishining matematik modeli $T \approx 6,5 \cos 30t + 21,5$ funksiyadir.

Mashqlar

127. Antarktidadagi Qutb bazasida 30 yil mobaynida o‘rtacha temperatura quyidagicha bo‘lganligi ma’lum:

Oyning tartib raqami	1	2	3	4	5	6	7	8	9	10	11	12
Temperatura (°C)	0	–4	–10	–15	–16	–17	–18	–19	–17	–13	–6	–1

O‘rtacha temperatura o‘zgarishining matematik modelini tuzing.

128. Dengiz qirg‘og‘ida dengiz suvining ko‘tarilishi va orqaga qaytishi jarayoni kuzatilganda quyidagilar aniqlandi: 1) suv chuqurligining eng katta va eng kichik qiymatlari orasidagi farq 14 metr; 2) suv chuqurligi eng katta qiymatlarga o‘rtacha har 12,4 soatda erishadi. Suv chuqurligining vaqtga nisbatan o‘zgarishining matematik modelini tuzing va uni grafik ko‘rinishda ifodalang.

129. Velosiped g‘ildiragida sariq rangli nurqaytargich o‘rnatilgan. Velosiped kechasi tekis yo‘l bo‘ylab harakatlanganda u videotasvirga olindi. Videotasvir asosida nurqaytargichning yo‘lga nisbatan balandligi vaqt o‘tishi bilan qanday o‘zgargani aniqlanib, quyidagi jadval to‘ldirildi:

Vaqt (t, s)	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
Balandlik (H, sm)	19	17	38	62	68	50	24	15	31

a) Sinus funksiyasidan foydalanib, jarayonning matematik modelini tuzing;

b) jarayonning grafik ko‘rinishini keltiring;

c) g‘ildirakning radiusni toping;

d) velosiped qanday tezlikda harakatlanmoqda?

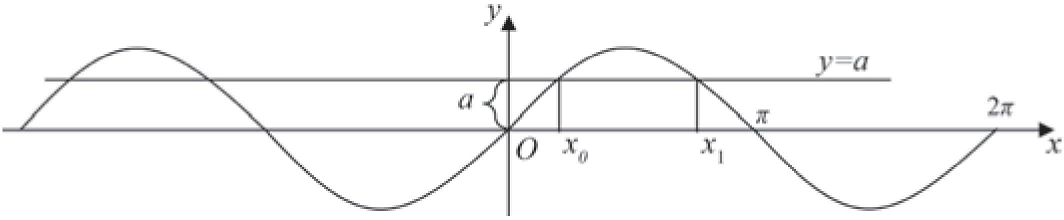
59-61 ENG SODDA TRIGONOMETRIK TENGLAMALAR

$\sin x = a$ tenglama

Ma’lumki, $-1 \leq \sin x \leq 1$, shuning uchun $\sin x = a$ tenglama $|a| > 1$ bo‘lganida yechimga ega emas. $-1 \leq a \leq 1$ oraliqda tenglamaning yechimini topish uchun quyidagi ta’rifni kiritamiz.

$a \in [-1;\ 1]$ sonning ***arksinusi*** deb sinusi a ga teng bo‘lgan $x \in [-\frac{\pi}{2}; \frac{\pi}{2}]$ songa aytiladi: agar $\sin x = a$ va $x \in [-\frac{\pi}{2}; \frac{\pi}{2}]$ bo‘lsa, $\arcsin a = x$.

Tenglamani yechish uchun 16- rasmdagi $y = \sin x$ funksiya grafigidan foydalanamiz.



16- rasm.

Grafikdan ko‘rinadiki, $a \in [-1;\ 1]$ bo‘lganda $y = a$ funksiya $[0;\ 2\pi]$ oraliqda $y = \sin x$ funksiya grafigini abssissalari x_0 va $x_1 = \pi - x_0$ bo‘lgan nuqtalarda kesadi. Bu ikki nuqtani bitta formula orqali yozish mumkin:

$$x = (-1)^n \arcsin a, \text{ bu yerda } n = 0, 1.$$

$y = \sin x$ funksiyaning davriyligidan foydalanib, tenglamani yechish uchun ushbu formulani hosil qilamiz:

$$x = (-1)^k \arcsin a + \pi k,\ k \in Z. \qquad (1)$$

1- misol. Hisoblang: 1) $\arcsin \frac{\sqrt{3}}{2}$; 2) $\arcsin\left(-\frac{1}{2}\right)$.

△ Ta’rifga ko‘ra $-1 \le \frac{\sqrt{3}}{2} \le 1$, $\frac{\pi}{3} \in [-\frac{\pi}{2}; \frac{\pi}{2}]$ va $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ bo‘lgani uchun $\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$. Huddi shuningdek, $\arcsin(-\frac{1}{2}) = -\arcsin \frac{1}{2} = -\frac{\pi}{6}$ bo‘ladi. ▲

2- misol. Tenglamani yeching: $\sin x = \frac{1}{2}$.

△ (1) Formulaga ko‘ra tenglamaning yechimi

$x = (-1)^k \arcsin \frac{1}{2} + \pi k = (-1)^k \frac{\pi}{6} + \pi k,\ \ k \in Z$ bo‘ladi. ▲

3-misol. Tenglamani yeching: $\sin\left(\frac{\pi}{12} - \frac{x}{2}\right) = \frac{\sqrt{2}}{2}$.

△ $y = \sin x$ funksiya toq bo‘lgani uchun $\sin\left(\frac{x}{2} - \frac{\pi}{12}\right) = -\frac{\sqrt{2}}{2}$ bo‘ladi.

(1) formulani qo‘llab, $\frac{x}{2} - \frac{\pi}{12} = (-1)^k \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \pi k,\ \ k \in Z$ tenglikni ho-

sil qilamiz. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$ bo‘lgani uchun $\frac{x}{2}-\frac{\pi}{12}=(-1)^k\left(-\frac{\pi}{4}\right)+\pi k$ $\frac{x}{2}-\frac{\pi}{12}=(-1)^k\left(-\frac{\pi}{4}\right)+\pi k$ yoki $x=\frac{\pi}{6}+(-1)^{k+1}\frac{\pi}{2}+2k\pi,\ k\in Z$ yechimlarni olamiz. ▲

sin$x=a$ tenglamaning xususiy hollardagi yechimlarini keltiramiz:

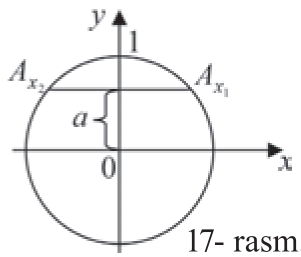
$a=1$ bo‘lganda $x=\frac{\pi}{2}+2\pi k,\ k\in Z$; $a=-1$ bo‘lganda $x=\frac{3}{2}\pi+2\pi k,\ k\in Z$;

$a=0$ bo‘lganda $x=\pi k,\ k\in Z$.

4- misol. Tenglamani yeching: $\sin\left(\frac{\pi}{10}-\frac{x}{2}\right)=0$.

△ $a=0$ ekanidan $-\frac{\pi}{10}+\frac{x}{2}=\pi k$, $\frac{x}{2}=\pi k+\frac{\pi}{10}$, ya’ni $x=\frac{\pi}{5}+2\pi k,\ k\in Z$ yechimlarni topamiz. ▲

sin$x=a$ tenglamani yechishni birlik doirada tushuntirish oson. sin$x$ ning ta’rifiga ko‘ra, uning qiymati birlik doiradagi A_x nuqtaning ordinatasidir. $|a|<1$ bo‘lganda bunday nuqtalar 2 ta, ya’ni A_{x_1} va A_{x_2}. $a=\pm1$ bo‘lganda esa 1 ta (17- rasm).



17- rasm.

cos$x=a$ tenglama

$-1\le\cos x\le1$ bo‘lgani uchun cos$x=a$ tenglama $|a|>1$ bo‘lganda yechimga ega emas. $-1\le a\le1$ oraliqda tenglamani yechish uchun ushbu ta’rifni kiritamiz.

$a\in[-1;\ 1]$ sonning ***arkkosinusi*** deb kosinusi a ga teng bo‘lgan $x\in[0;\ \pi]$ songa aytiladi: agar cos$x=a$ va $x\in[0;\ \pi]$ bo‘lsa, arccos $a=x$.

Ta’rifga ko‘ra, $[0;\ \pi]$ oraliqda cos$x=a$ tenglama bitta $x=\arccos a$ ildizga ega. $y=\cos x$ funksiya juft bo‘lganligi uchun $[-\pi;0]$ oraliqda ham bitta $x=-\arccos a$ yechimga ega. Funksiyaning davri 2π. U holda cos$x=a$ tenglamani yechish uchun ushbu formulani hosil qilamiz. $x=\pm\arccos a+2\pi k,\ k\in Z$ (2)

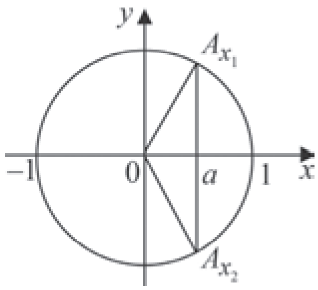
5- misol. Hisoblang: 1) $\arccos\frac{\sqrt{3}}{2}$; 2) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$.

△ Ta’rifga ko‘ra, $-1\le\frac{\sqrt{3}}{2}\le1$, $\frac{\pi}{6}\in[0;\pi]$ va $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ bo‘lgani uchun $\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{6}$ bo‘ladi. Huddi shuningdek, $\arccos\left(-\frac{\sqrt{2}}{2}\right)=\frac{3\pi}{4}$ bo‘ladi. ▲

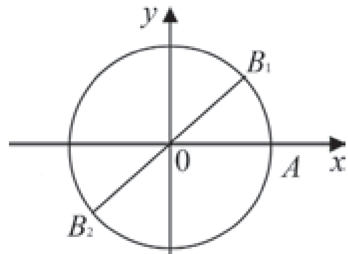
6- misol. Tenglamani yeching: $\cos x = \frac{\sqrt{3}}{2}$.

(2) formulaga ko‘ra tenglamaning yechimi $x = \pm \arccos\frac{\sqrt{3}}{2} + 2\pi k,\ k \in Z$, ammo $\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

Demak, yechim ushbu ko‘rinishda bo‘ladi: $x = \pm\frac{\pi}{6} + 2\pi k,\ k \in Z$



18- rasm.



19- rasm.

cos$x$=a tenglama yechilishini birlik doirada tushuntiramiz (18- rasm). cos$x$ funksiyaning ta’rifiga ko‘ra uning qiymati birlik doiradagi A_x nuqtaning abssissasi bo‘ladi. $|a|<1$ bo‘lganda bunday nuqtalar 2 ta, ya’ni A_{x_1} va A_{x_2}; $a=1$ va $a=-1$ bo‘lganda bunday nuqta bitta.

cos$x$=a tenglamaning xususiy hollardagi yechimlarini keltiramiz:

$a=1$ bo‘lganda $x=2\pi k, k\in Z$; $a=-1$ bo‘lganda $x=\pi+2\pi k, k\in Z$;

$a=0$ bo‘lganda $x = \frac{\pi}{2} + \pi k,\ k\in Z$.

7- misol. Tenglamani yeching: $\cos\left(3x - \frac{\pi}{4}\right) = 0$.

cos$x$=0 tenglamaning yechimi formulasidan $3x - \frac{\pi}{4} = \frac{\pi}{2} + \pi k$ ni hosil qilamiz. Bundan, $x = \frac{\pi}{4} + \frac{\pi k}{3},\ k \in Z$.

tg$x$=a tenglama

Bu tenglamani yechish uchun quyidagi ta’rifni kiritamiz. $a\in R$ sonning ***arktangensi*** deb, tangensi a songa teng bo‘lgan $x\in(-\pi/2;\pi/2)$ songa aytiladi: agar tg$x$=a va $x\in(-\pi/2;\pi/2)$ bo‘lsa, arctg $a=x$.

$\text{tg}x = \frac{\sin x}{\cos x}$ bo‘lgani uchun tg$x$ birlik doiradagi $B\ (x;\ y)$ nuqta ordinatasining abssissasiga nisbatiga teng (19- rasm), ya’ni bu nuqta $\frac{y}{x} = a$ to‘g‘ri chiziq bilan birlik doiraning kesishish nuqtasidir. 19- rasmga ko‘ra bunday nuqtalar 2 ta: B_1

va B_2 nuqtalar. Shuning uchun tenglamaning yechimi quyidagicha bo'ladi:

$$x = \mathrm{arctg}\, a + \pi n,\ n \in Z. \qquad (3)$$

8- misol. Hisoblang: 1) arctg1; 2) arctg($-\sqrt{3}$).

△ 1) $\mathrm{tg}\frac{\pi}{4} = 1$ va $\frac{\pi}{4} \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ bo'lgani uchun $\mathrm{arctg}\, 1 = \frac{\pi}{4}$;

2) $\mathrm{tg}\left(-\frac{\pi}{3}\right) = -\sqrt{3}$ va $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ bo'lgani uchun $\mathrm{arctg}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$. ▲

9- misol. Tenglamani yeching: $\mathrm{tg}\left(x - \frac{\pi}{6}\right) = -\sqrt{3}$.

△ (3) ga ko'ra, tenglamaning yechimlari quyidagicha bo'ladi:

$x - \frac{\pi}{6} = \mathrm{arctg}(-\sqrt{3}) + \pi n$. $\mathrm{arctg}(-\sqrt{3}) = -\mathrm{arctg}(\sqrt{3}) = -\frac{\pi}{3}$ bo'lgani uchun tenglamaning yechimlari $x - \frac{\pi}{6} = -\frac{\pi}{3} + \pi n$, yoki $x = -\frac{\pi}{6} + \pi n,\ n \in Z$. ▲

Eng sodda trigonometrik tenglamalar uchun jadvalni keltramiz:

Tenglama	Yechimlari	Ba'zi xossalar
$\sin x = a$	$x = (-1)^k \arcsin a + \pi k,\ k \in Z.$	$\arcsin(-a) = -\arcsin a,\ \lvert a \rvert \leqslant 1.$
$\cos x = a$	$x = \pm\arccos a + 2\pi k,\ k \in Z.$	$\arccos(-a) = \pi - \arccos a,\ \lvert a \rvert \leqslant 1.$
$\mathrm{tg}\, x = a$	$x = \mathrm{arctg}\, a + \pi k,\ k \in Z.$	$\mathrm{arctg}(-a) = -\mathrm{arctg}\, a,\ a \in R.$

Uchinchi ustunda keltirilgan xossalar manfiy sonlar arksinuslari (arkkosinuslari, arktangenslari) qiymatlarini musbat sonlar arksinuslari qiymatlari orqali topish imkoniyatini beradi. Masalan, $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\arcsin\frac{\sqrt{2}}{2} = -\frac{\pi}{4}$,

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \arccos\frac{\sqrt{3}}{2} = \pi - \frac{\pi}{6} = \frac{5\pi}{6},$$

$$\mathrm{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\mathrm{arctg}\frac{\sqrt{3}}{3} = -\frac{\pi}{6}.$$

10- misol. Tenglamani yeching: $\cos(10x + \frac{\pi}{8}) = \frac{1}{2}$.

△ $10x + \frac{\pi}{8} = z$ belgilash kiritib, $\cos z = \frac{1}{2}$ tenglamani hosil qilamiz. Bundan (2) formulaga ko'ra $z = \pm\frac{\pi}{3} + 2\pi k,\ k \in Z$, ya'ni $10x + \frac{\pi}{8} = \pm\frac{\pi}{3} + 2\pi k$ yoki

$x=\frac{1}{10}\left(-\frac{\pi}{8}\pm\frac{\pi}{3}+2\pi k\right),\ k\in Z.$ ▲

$\sin x=\sin a$, $\cos x=\cos b$, $\mathrm{tg}x=\mathrm{tg}c$ ko'rinishdagi tenglamalar

Bunday tenglamalarning yechimi, mos ravishda, quyidagicha bo'ladi:

$$x=(-1)^k a+\pi k,\quad k\in Z;\quad x=\pm b+2\pi n,\ n\in Z;\quad x=c+\pi m,\ m\in Z. \qquad (4)$$

11- misol. Tenglamani yeching: $\cos(3x-40°)=\cos(2x+60°)$.

△(4) formulaga ko'ra, $3x-40°=\pm(2x+60°)+360°n,\ n\in Z$ tenglamani hosil qilamiz. Bundan noma'lum x topiladi:

$$3x-40°=2x+60°+360°n \iff x=100°+360°n,\ n\in Z;$$

$$3x-40°=-2x-60°+360°n,\ 5x=-20°+360°n \iff x=-4°+72°n,\ n\in Z.$$ ▲

12- misol. Tenglamani yeching: $\sin^2 x+3\sin x+2=0$.

△$\sin x=z$ belgilash kiritib, $z^2+3z+2=0$ kvadrat tenglamaga kelamiz. Bu tenglamani yechib $z_1=-2,\ z_2=-1$ lar topiladi. Belgilashga ko'ra $\sin z=-2$ va $\sin x=-1$ tenglamalarni hosil qilamiz. $\sin z=-2$ yechimga ega emas. $\sin x=-1$ tenglama $x=270°+360°k,\ k\in Z$ yechimga ega. Demak, tenglamaning yechimi $x=270°+360°k,\ k\in Z$ bo'ladi. ▲

Savol va topshiriqlar

1. $\sin x=a$ tenglama qanday yechiladi? Misolda tushuntiring.
2. $\cos x=a$ tenglama qanday yechiladi? Misol keltiring.
3. $\mathrm{tg}x=a$ tenglama qanday yechiladi? Misol yordamida tushuntiring.
4. arcsin a soniga ta'rif bering. Misolda tushuntiring.
5. arccos a soniga ta'rif bering. Misolda tushuntiring.
6. arctg a soniga ta'rif bering. Misolda tushuntiring.

Mashqlar

Hisoblang (**130–141**):

130. 1) $\arcsin 0$; 2) $\arcsin\frac{\sqrt{3}}{2}$; 3) $\arcsin\frac{1}{2}$; 4) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

131. 1) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$; 2) $\arcsin\left(-\frac{1}{2}\right)$; 3) $\arcsin 1$; 4) $\arcsin(-1)$.

132. 1) $\arccos 0$; 2) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$; 3) $\arccos\frac{\sqrt{2}}{2}$; 4) $\arccos(-1)$.

133. 1) $\arccos\left(-\frac{1}{2}\right)$; 2) $\arccos\frac{1}{2}$; 3) $\arccos 1$; 4) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$.

134. 1) $\mathrm{arctg}\,1$; | 2) $\mathrm{arctg}\left(-\frac{1}{\sqrt{3}}\right)$; | 3) $\mathrm{arctg}\frac{1}{\sqrt{3}}$; | 4) $3\,\mathrm{arctg}\left(-\sqrt{3}\right)$.

135. 1) $\mathrm{arctg}\,0$; | 2) $\mathrm{arctg}\left(-\sqrt{3}\right)$; | 3) $\mathrm{arctg}(-1)$; | 4) $7\,\mathrm{arctg}\left(-\frac{1}{\sqrt{3}}\right)$.

136. 1) $\arcsin 1+\arcsin(-1)$; | 2) $2\arcsin\frac{\sqrt{3}}{2}+4\arcsin\frac{1}{2}$.

137. 1) $4\arcsin\frac{\sqrt{2}}{2}-2\arcsin\left(-\frac{\sqrt{2}}{2}\right)$; | 2) $\arcsin\left(-\frac{1}{2}\right)+\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

138. 1) $2\arccos 1+3\arccos 0$; | 2) $6\arccos\frac{\sqrt{3}}{2}-3\arccos\left(-\frac{1}{2}\right)$.

139. 1) $2\arccos(-1)-3\arccos 0$; | 2) $2\arccos\left(-\frac{\sqrt{2}}{2}\right)+4\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

140. 1) $3\,\mathrm{arctg}\sqrt{3}+3\arccos\frac{1}{2}$; | 2) $3\,\mathrm{arctg}\left(-\frac{1}{\sqrt{3}}\right)+2\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

141. 1) $2\,\mathrm{arctg}\,1+3\arcsin\left(-\frac{1}{2}\right)$; | 2) $5\,\mathrm{arctg}\,(-\sqrt{3})-3\arccos\left(-\frac{\sqrt{2}}{2}\right)$.

Ifodalar ma'noga ega yoki ega emasligini aniqlang (**142–143**):

142. 1) $\arccos(\sqrt{8}-3)$; | 2) $\arcsin(2-\sqrt{15})$; | 3) $\arccos(3-\sqrt{18})$.

143. 1) $\mathrm{tg}(2\arcsin\frac{\sqrt{2}}{2})$; | 2) $\arcsin(\sqrt{6}-2)$; | 3) $\mathrm{tg}(3\arccos\frac{1}{2})$.

Tenglamani yeching (**144–161**):

144. 1) $\sin x=-\frac{1}{2}$; | 2) $\sin x=\frac{\sqrt{3}}{2}$; | 3) $\sin x=-\frac{\sqrt{2}}{2}$; | 4) $\sin 2x=\frac{1}{2}$.

145. 1) $\sin x=\frac{\sqrt{2}}{2}$; | 2) $\sin x=1$; | 3) $\sin x=-\frac{\sqrt{3}}{2}$; | 4) $\sin 2x=\frac{\sqrt{3}}{2}$.

146. 1) $\cos x=-\frac{\sqrt{2}}{2}$; | 2) $\cos x=\frac{\sqrt{3}}{2}$; | 3) $\cos 2x=-1$; | 4) $\cos 3x=1$.

147. 1) $\cos x=\frac{1}{2}$; | 2) $\cos x=-1$; | 3) $\cos 5x=-\frac{1}{2}$; | 4) $\cos 3x=-1$.

148. 1) $\operatorname{tg}x=-\sqrt{3}$; | 2) $\operatorname{tg}x=1$; | 3) $\operatorname{tg}9x=-1$; | 4) $\operatorname{tg}3x=\frac{\sqrt{3}}{3}$.

149. 1) $\operatorname{tg}x=0$; | 2) $\operatorname{tg}x=2$; | 3) $\operatorname{tg}6x=-3$; | 4) $\operatorname{tg}5x=-\frac{\sqrt{3}}{3}$.

150. 1) $2\cos x+1=0$; | 2) $2\cos x-\sqrt{3}=0$; | 3) $2\cos x-\sqrt{2}=0$.

151. 1) $\sqrt{2}\sin x-1=0$; | 2) $2\sin x+\sqrt{3}=0$; | 3) $2\sin x+\sqrt{2}=0$.

152. 1) $\sin\left(-\frac{x}{2}\right)=\frac{\sqrt{3}}{2}$; | 2) $\operatorname{tg}4x=-\frac{1}{\sqrt{3}}$; | 3) $\cos(-3x)=\frac{\sqrt{2}}{2}$.

153. 1) $2\sin\left(2x+\frac{\pi}{4}\right)=-\sqrt{2}$; | 2) $\sqrt{3}\operatorname{tg}\left(\frac{x}{2}-\frac{\pi}{3}\right)=1$; | 3) $2\cos(\frac{x}{3}-\frac{\pi}{6})=\sqrt{3}$.

154. 1) $\cos\left(\frac{\pi}{3}-2x\right)=-1$; | 2) $\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{3}\right)=1$; | 3) $2\cos\left(\frac{\pi}{6}-\frac{x}{2}\right)=\sqrt{3}$.

155. 1) $2\sin\left(\frac{\pi}{6}-\frac{x}{2}\right)=\sqrt{3}$; | 2) $2\cos\left(\frac{\pi}{4}-3x\right)=\sqrt{2}$; | 3) $\sin\left(x+\frac{\pi}{4}\right)=1$.

156. 1) $(2\sin x+\sqrt{2})(\sin 4x+1)=0$; | 2) $(2-\cos x)(1+3\cos x)=0$.

157. 1) $2\sin^2 x-\sin x-1=0$; | 2) $4\cos^2 x-8\cos x-3=0$;
3) $2\sin^2 x-\sin x-6=0$; | 4) $2\cos^2 x-\cos x-6=0$.

158. 1) $2\sin^2 x-\sin x-1=0$; | 2) $4\cos^2 x-8\cos x-3=0$;
3) $2\sin^2 x-\sin x-6=0$; | 4) $2\cos^2 x-\cos x-6=0$.

159. 1) $2\cos^2 x-\sin x+1=0$; | 2) $\operatorname{tg}^2 x-3\operatorname{tg}x-4=0$;
3) $4\sin^2 x-\cos x-1=0$; | 4) $\operatorname{tg}x-\sqrt{3}\operatorname{tg}x+1=\sqrt{3}$.

160. 1) $\cos x=\cos 2x$; | 2) $\operatorname{tg}2x=\operatorname{tg}3x$; | 3) $\sin 7x=\sin 3x$; | 4) $\cos 4x=\cos 5x$.

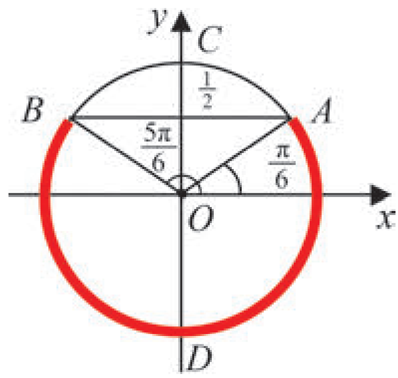
161. 1) $\sin 4x=\sin x$; | 2) $\sin 2x=\cos 3x$; | 3) $\operatorname{tg}10x=\operatorname{tg}8x$; | 4) $\sin 5x=\sin 7x$.

ENG SODDA TRIGONOMETRIK TENGSIZLIKLAR

$a_1 < \sin x < b_1$, $a_2 < \cos x < b_2$, $a_3 < \mathrm{tg}x < b_3$ ko'rinishdagi tengsizliklar *eng sodda trigonometrik tengsizliklar* deyiladi. Bu yerda a_1, b_1, a_2, b_2, a_3, b_3 – berilgan haqiqiy sonlar. Bunday tengsizliklarni yechishda birlik doiradan, funksiya grafigidan foydalanish qulay.

1- misol. $\sin x \le 0{,}5$ tengsizlikni $[0,\ 2\pi]$ kesmada yeching.

△Birlik doirani qaraymiz. Bu doirada ordinatalari 0,5 ga teng va undan kichik nuqtalarni topamiz. 20- rasmdan ravshanki, *BDA* yoyning barcha nuqtalari yuqoridagi shartni qanoatlantiradi. Shuning uchun *x* sonlarning $\left[0;\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6};2\pi\right]$ to'plami tengsizlikning yechimi bo'ladi. *Javob*: $x\in\left[0;\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6};2\pi\right]$ ▲

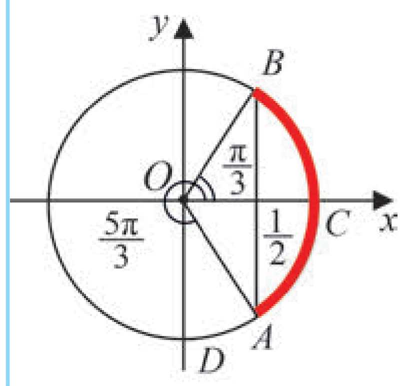


20- rasm.

2- misol. $\cos x > \frac{1}{2}$ tengsizlikni $[0,\ 2\pi]$ kesmada yeching.

△Birlik doirada abssissalari $\frac{1}{2}$ ga teng va undan katta nuqtalarni topamiz. 21- rasmdan ko'rinb turibdiki, *ACB* yoyning barcha nuqtalari yuqoridagi shartni qanoatlantiradi. Shuning uchun *x* larning $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right]$ to'plami tengsizlikning yechimi bo'ladi.

Javob: $x\in\left[0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right]$ ▲



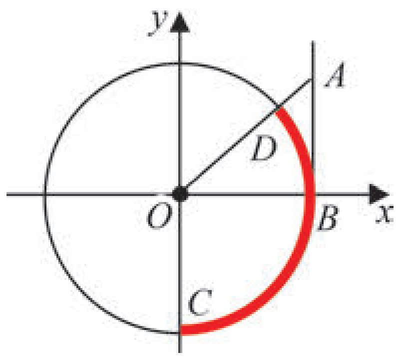
21- rasm.

3- misol. $\mathrm{tg}x \le 1$ tengsizlikni $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda yeching.

△Birlik doiraning *B* nuqtasidan *Oy* o'qiga parallel *AB* to'g'ri chiziq o'tkazamiz (22- rasm).

Unda A nuqtani shunday tanlaymizki, bunda $OB=AB$ bo‘lsin. $\triangle AOB$ teng yonli va to‘g‘ri burchaklidir. OA gipotenuzaning aylana bilan kesishuv nuqtasi D bo‘lsin. Rasmdan ravshanki, DBC yoyning barcha nuqtalari berilgan tengsizlikni qanoatlantiradi.

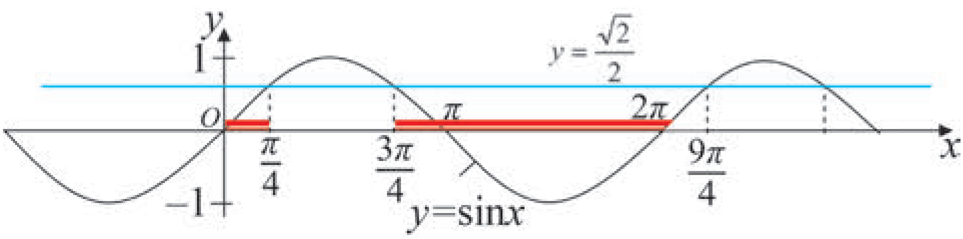
Javob: $x\in\left(-\frac{\pi}{2};\frac{\pi}{4}\right]$. ▲



22- rasm.

4- misol. Tengsizlikni yeching: $\sin x<\frac{\sqrt{2}}{2}$.

△ Bitta koordinatalar sistemasiga $y=\sin x$ va $y=\frac{\sqrt{2}}{2}$ (23- rasm) funksiyalar



23- rasm.

grafiklarini chizib, $\sin x=\frac{\sqrt{2}}{2}$ tenglamaning $[0;\ 2\pi]$ kesmadagi yechimini topamiz. Rasmdan ko‘rinadiki, $\sin x<\frac{\sqrt{2}}{2}$ tengsizlikning $[0;\ 2\pi]$ kesmadagi yechimi $\left[0;\frac{\pi}{4}\right)$ va $\left(\frac{3\pi}{4};2\pi\right]$ oraliqlar bo‘ladi. Funksiyaning davriyligidan x ning $\left[2\pi n;\frac{\pi}{4}+2\pi n\right)\cup\left(\frac{3\pi}{4}+2\pi n;2\pi(n+1)\right]$, $n\in Z$ to‘plami tengsizlikning yechimi bo‘ladi. ▲

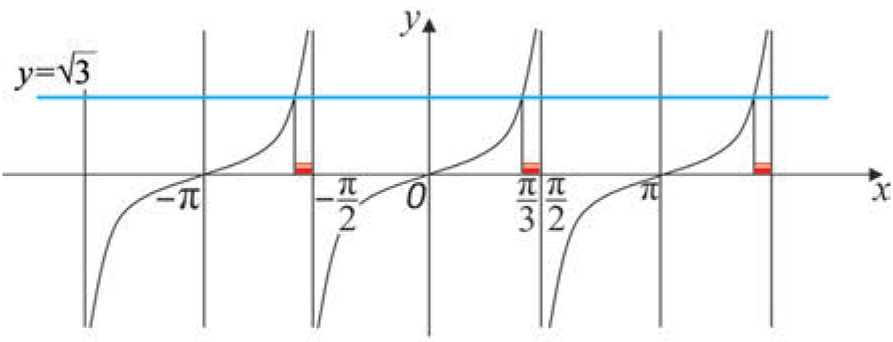
5- misol. Tengsizlikni yeching: $-2\cos x\geq 1$. Mos rasm chizing.

△ Avval $y=\cos x$ va $y=-\frac{1}{2}$ funksiyalar grafigini bitta koordinatalar sistemasiga chizamiz. Undan $\cos x=-\frac{1}{2}$ tenglamaning $[0;\ 2\pi]$ kesmadagi yechimlari $\frac{2\pi}{3}$ va $\frac{4\pi}{3}$ ekanini aniqlaymiz. Demak, tengsizlikning yechimlari $\left[\frac{2\pi}{3}+2\pi n;\frac{4\pi}{3}+2\pi n\right]$, $n\in Z$ kesmalardan iborat ekan.▲

6- misol. Tengsizlikni yeching: $\operatorname{tg} x \geq \sqrt{3}$.

△ $y=\operatorname{tg} x$ va $y=\sqrt{3}$ funksiyalar grafigini bitta koordinatalar sistemasida chizamiz (24- rasm). $\operatorname{tg} x=\sqrt{3}$ tenglamani [0; π] kesmadagi yechimini topamiz. Bu tenglamaning yechimi $x = \frac{\pi}{3}$. Shuning uchun tengsizlikning [0; π] kesmadagi yechimlari to'plami $\left[\frac{\pi}{3};\frac{\pi}{2}\right)$ oraliqdir. $y=\operatorname{tg}x$ funksiyaning davri π ekanidan foydalanib, tengsizlikning barcha yechimlarini topamiz:

$$\left[\frac{\pi}{3}+\pi n;\frac{\pi}{2}+\pi n\right),\ n\in Z.$$ ▲



24- rasm.

Savol va topshiriqlar

$\sin x > \frac{\sqrt{3}}{2}$, $\cos x > -\frac{\sqrt{3}}{2}$, $\operatorname{tg}x > -1$ tengsizliklar qanday yechiladi?

Mashqlar

162. Tengsizlikni berilgan oraliqda yeching:

1) $\sin x > \frac{1}{2},\ x\in[0;\pi]$;

2) $\cos x > \frac{\sqrt{2}}{2},\ x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;

3) $\operatorname{tg} x > -\sqrt{3}, x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$;

4) $\cos x > \frac{1}{2},\ x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;

5) $\sin x \le \frac{\sqrt{3}}{2},\ x \in [-\pi; 0]$;

6) $\operatorname{tg} x < \frac{1}{\sqrt{3}},\ x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$;

7) $\cos x < -\frac{\sqrt{3}}{2},\ x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$;

8) $\cos 2x \le \frac{\sqrt{2}}{2},\ x \in \left[\frac{\pi}{2}; \frac{3\pi}{2}\right]$.

Tengsizlikni yeching (**163–169**):

163. 1) $\sin x \ge \frac{\sqrt{2}}{2}$; 2) $\mathrm{css}\, x < -\frac{\sqrt{2}}{2}$; 3) $\operatorname{tg} x < -\frac{1}{\sqrt{3}}$; 4) $\sin x < -\frac{\sqrt{3}}{2}$.

164. 1) $\sin x > \frac{1}{2}$; 2) $\operatorname{tg} x > -1$; 3) $\cos x \le -\frac{\sqrt{2}}{2}$; 4) $\cos x \le \frac{1}{2}$.

165. 1) $\sin 3x < \frac{1}{2}$; 2) $\sin \frac{x}{4} < -\frac{\sqrt{3}}{2}$; 3) $\cos \frac{x}{2} > \frac{\sqrt{3}}{2}$; 4) $\operatorname{tg} 3x > 1$.

166. 1) $2\cos(2x + \frac{\pi}{3}) \le \sqrt{2}$; 2) $\sqrt{2}\sin\left(\frac{x}{2} + \frac{\pi}{6}\right) \ge 1$; 3) $2\cos(2x - \frac{\pi}{3}) > \sqrt{3}$.

167. 1) $\sin 2x \cos\frac{\pi}{3} - \cos 2x \sin\frac{\pi}{3} \le \frac{\sqrt{3}}{2}$; 2) $2\sin 2x \cos 2x \ge \frac{1}{2}$.

168. 1) $\sin\frac{\pi}{4}\cos 3x + \cos\frac{\pi}{4}\sin 3x < \frac{\sqrt{2}}{2}$; 2) $\cos\frac{\pi}{4}\cos 2x - \sin 2x \sin\frac{\pi}{4} < -\frac{\sqrt{3}}{2}$.

169. 1) $\cos\left(\frac{x}{2} + 1\right) \ge \frac{1}{2}$; 2) $\sin\left(\frac{x}{4} - 2\right) < \frac{\sqrt{2}}{2}$; 3) $\cos\left(1 - \frac{x}{3}\right) \ge \frac{\sqrt{2}}{2}$.

Nazorat ishi namunasi

Tenglamalarni yeching (**1–4**):

1. $\sin 3x = 0$.
2. $4\cos 6x = -2\sqrt{3}$.
3. $5 \cdot \operatorname{tg} 4x = 3$.
4. $5\operatorname{tg}^2 x - 4\operatorname{tg} x - 1 = 0$.

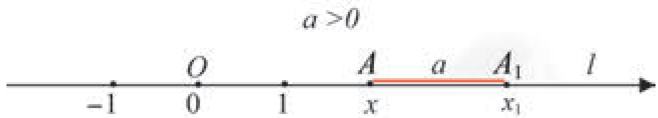
Tengsizliklarni $x \in [0; \pi]$ oraliqda yeching (**5–6**):

5. $\sin x > \frac{1}{2}$.
6. $\operatorname{tg} x \le -1$.

68 GRAFIKLARNI ALMASHTIRISH

Siljitish

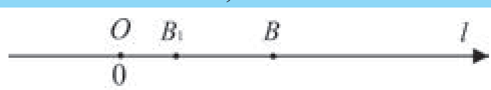
l son o‘qi va *O* nuqta undagi hisob boshi bo‘lsin (25- rasm). *l* ning har qaysi nuqtasi *a* birlik siljitilsin. Agar $a>0$ bo‘lsa, siljitish musbat yo‘nalishda (o‘q yo‘nalishida) bo‘ladi. Agar $a<0$ bo‘lsa, siljitish qarama-qarshi yo‘nalishda bajariladi, $a=0$ da nuqtalar o‘z joyidan siljimaydi. Agar *x* koordinatali $A=A(x)$ nuqta *a* birlikka siljitilganda $A_1(x_1)$ nuqtaga o‘tgan bo‘lsa, A_1 nuqtaning koordinatasi $x_1=x+a$ formula bo‘yicha aniqlanadi. *A* nuqta A_1 nuqtaning asli (proobrazi), A_1 esa *A* ning nusxasi (obrazi) deyiladi.



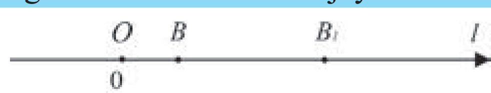
25- rasm.

Cho‘zish

l son o‘qida $B(x)$ nuqta *O* koordinata boshidan *k* marta uzoqlashtirilib (yoki *O* ga yaqinlashtirilib), $B_1(x)$ nuqtaga o‘tkazilgan bo‘lsin. B_1 nuqtaning koordinatasi $x_1=kx$ formula bo‘yicha hisoblanadi. Agar $k>0$ bo‘lsa, B_1 va *B* nuqtalar *O* nuqtaning bir tomonida; agar $k<0$ da B_1 va *B* nuqtalar *O* ning turli tomonida joylashadi. Agar $|k|<1$ bo‘lsa (26- rasm), $x=OB$ kesma *k* marta qisqaradi; agar $|k|>1$ bo‘lsa (27- rasm), *OB* kesma *k* marta cho‘ziladi, $k=1$ da *B* va B_1 nuqtalar ustma-ust tushadi, $k=-1$ da ular *O* nuqtaga nisbatan simmetrik joylashadi.



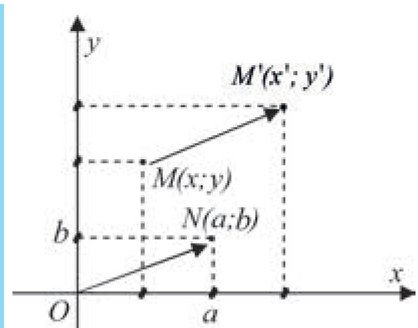
$0<k<1$ 26- rasm.



$k>1$ 27-rasm.

Parallel ko‘chirish

Parallel ko‘chirishda *xOy* koordinata tekisligidagi barcha nuqtalar bir xil yo‘nalishda bir xil masofaga ko‘chadi (28- rasm). Chunonchi, $O(0;\ 0)$ koordinata boshi $N(a;\ b)$ nuqtaga ko‘chirilgan bo‘lsa, $M(x;\ y)$ nuqta $M'(x';\ y')$ ga ko‘chadi. $M'(x';\ y')$ nuqtaning koordinatalari uchun quyidagi formula o‘rinli: $x'=x+a$, $y'=y+b$.



28- rasm.

Funksiya grafigini almashtirish

Yuqoridagi almashtirishlar (siljitish, cho'zish, parallel ko'chirish) $y=f(x)$ funksiya grafigi yordamida $y=f(x-a)+b$, $y=m\cdot f\left(\frac{x}{k}\right)$ (bunda a, b, m, k – o'zgarmas sonlar va $m\neq 0$, $k\neq 0$) funksiyalar grafigini chizish imkonini beradi.

Masalan, $y=f(x-a)+b$ funksiya grafigini $y=f(x)$ funksiya grafigi yordamida chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasi a birlik o'ngga siljitiladi va b birlik yuqoriga ko'tariladi, ya'ni $(a; b)$ vektor bo'yicha parallel ko'chiriladi.

$y=f(x)$ funksiya grafigi yordamida $y=m\cdot f\left(\frac{x}{k}\right)$ funksiya grafigini chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasining abssissasi Ox bo'ylab k marta siqiladi ($k>0$ bo'lsa – o'ngga, $k<0$ bo'lsa – chapga) va ordinatasi Oy o'q bo'ylab m birlik cho'ziladi ($m>0$ bo'lsa – yuqoriga, $m<0$ bo'lsa – pastga).

1- misol. $y=3x$ funksiya grafigi yordamida $y=3(x-1)+4$ funksiya grafigini chizing.

△ $y=3(x-1)+4$ funksiya grafigini chizish uchun $y=3x$ funksiya grafigi (1; 4) vektor bo'yicha parallel ko'chiriladi. ▲

2- misol. $y=-2x+4$ funksiya grafigi yordamida $y=-2(x+3)+5$ funksiya grafigini chizing.

△ $y=-2(x+3)+5$ funksiya grafigini chizish uchun $y=-2x+4$ funksiya grafigi (3; 1) vektor bo'yicha parallel ko'chiriladi. ▲

3- misol. $y=x^2$ parabola grafigidan foydalanib $y=2-(x+3)^2$ funksiya grafigini chizing.

△ $y=2-(x+3)^2$ funksiya grafigini chizish uchun $y=x^2$ funksiya grafigi avval 3 birlik chapga siljitiladi va Ox o'qiga nisbatan simmetrik ko'chiriladi. So'ngra hosil bo'lgan grafik Oy o'qi bo'yicha 2 birlik yuqoriga ko'tariladi. ▲

4- misol. $y=\sin x$ funksiya grafigi yordamida $y=\sin 2x$ funksiya grafigini chizing.

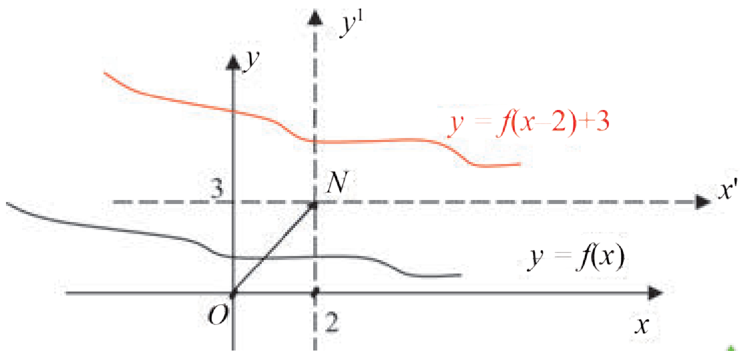
△ $y=\sin 2x$ funksiya grafigini chizish uchun $y=\sin x$ funksiya grafigining har bir nuqtasining abssissasi Ox o'qi bo'ylab ikki marta o'ngga siqiladi. ▲

5- misol. $y=\cos x$ funksiya grafigi yordamida $y=-2\cos\left(2x-\frac{\pi}{4}\right)$ funksiya grafigini chizing.

△ $y=-2\cos\left(2x-\frac{\pi}{4}\right)$ yoki $y=-2\cos 2\left(x-\frac{\pi}{8}\right)$ funksiya grafigini chizish uchun avval $y=\cos x$ funksiya grafigi o'ngga $\frac{\pi}{8}$ ga siljitiladi, keyin abssissasi o'ngga ikki marta siqiladi, ordinatasi ikki marta yuqoriga cho'ziladi. Hosil bo'l-gan grafik Ox o'qiga nisbatan simmetrik ko'chiriladi. ▲

6- misol. $y=f(x)$ funksiya grafigi yordamida $y=f(x-2)+3$ funksiya grafigini chizing.

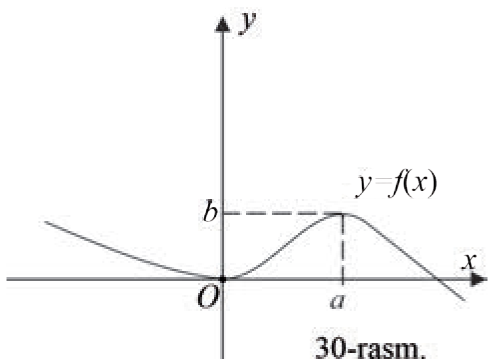
△ $y=f(x-2)+3$ funksiya grafigini chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasi (2; 3) vektor bo'yicha parallel ko'chiriladi (29- rasm). ▲



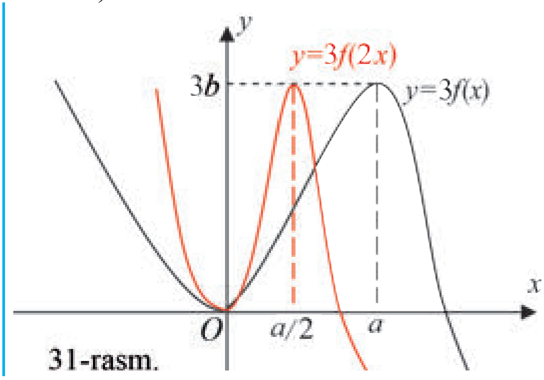
29- rasm. ▲

7- misol. $y=f(x)$ funksiya grafigi yordamida (30- rasm) $y=3f(2x)$ funksiya grafigini chizing ($m=3,\quad k=\frac{1}{2}$ bo'lgan hol).

△ $y=f(x)$ funksiya grafigi Ox o'q bo'ylab o'ngga 2 marta siqiladi va Oy o'q bo'ylab yuqoriga 3 marta cho'ziladi (31- rasm). ▲



30-rasm.



31-rasm.

Savol va topshiriqlar

1. Siljitish nima? Cho'zish-chi? Parallel ko'chirish-chi? Misollar keltiring.

2. $y=\sin x$ funksiya grafigi yordamida $y=-\sin\left(x-\frac{\pi}{3}\right)$ funksiya grafigini chizing.

Mashqlar

170. $y=f(x)=x^2-2x+3$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=f(x)+1$; 2) $y=3f(x)$; 3) $y=3f(x)-2$;

4) $y=f(x-1)+1$; 5) $y=2f(x+1)+1$; 6) $y=f\left(\frac{x}{2}\right)$;

7) $y=\frac{1}{2}f(2x)$; 8) $y=f(2x)-3$; 9) $y=2f(2x)-5$.

171. $y=f(x)=x^2-5x+6$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=f(x-1)$; 2) $y=f\left(\frac{x}{3}\right)$; 3) $y=f(2x)$; 4) $y=3f\left(\frac{x}{3}\right)+1$;

5) $y=-f(x)$; 6) $y=2f(x)-3$; 7) $y=-f(-x)$; 8) $y=2f(x+1)+5$.

172. $y=\cos x$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=\cos x-1$; 2) $y=2\cos x+1$;

3) $y=-\cos\left(x+\frac{\pi}{4}\right)$; 4) $y=3\cos\left(2x-\frac{\pi}{3}\right)$.

69-70 PARAMETRIK KO‘RINISHDA BERILGAN SODDA FUNKSIYALARNING GRAFIKLARI

Moddiy nuqtaning $(x; y)$ koordinatlari t parametrga bog‘liq bo‘lsin: $x=\varphi(t)$, $y=\psi(t)$. t biror T oraliqda o‘zgarganda $(\varphi(t), \psi(t))$ nuqtalar to‘plami qanday bo‘ladi? Bu to‘plamni *parametrik ko‘rinishda berilgan funksiyaning grafigi* deb ataymiz.

1- misol. Moddiy nuqtaning koordinatalari parametrik ko‘rinishda $\begin{cases} x=3t+1, \\ y=5t+8 \end{cases}$ berilgan. Bu moddiy nuqta harakati davomida chizgan chiziqni (moddiy nuqta trayektoriyasini) toping.

△ Tenglamalardan t parametrni topamiz: $t=\frac{x-1}{3}$ va $t=\frac{y-8}{5}$.

Hosil bo‘lgan ifodalardan $\frac{x-1}{3}=\frac{y-8}{5}$ tenglamaga kelamiz. Bundan $5x-5=3y-24$ yoki $5x-3y+19=0$. Bu to‘g‘ri chiziqning tenglamasidir.

Demak, izlangan funksiya $3y=5x+19$ yoki $y=\frac{5}{3}x+\frac{19}{3}$ ekan.

Javob: $y=\frac{5}{3}x+\frac{19}{3}$. ▲

2- misol. $\begin{cases} x=3+5\sin t, \\ y=-7+5\cos t \end{cases}$ parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziq bo‘ladi?

△Berilgan tengliklardan $\sin t=\frac{x-3}{5}$, $\cos t=\frac{y+7}{5}$ ekanini topamiz.

$\sin^2 t+\cos^2 t=1$ ayniyatdan foydalanib, $\left(\frac{x-3}{5}\right)^2+\left(\frac{y+7}{5}\right)^2=1$ tenglamaga kelamiz. Bundan $(x-3)^2+(y+7)^2=25$. Bu tenglama markazi $(3;\ -7)$ va radiusi $r=5$ bo‘lgan aylana tenglamasidir.▲

3- misol. Moddiy nuqta koordinatalari $x=7t^2+1$, va $y=3t$ qonuniyat bilan o‘zgarsa, x va y orasidagi bog‘lanishni aniqlang, $t\geq 0$.

△Berilgan qonuniyatlardan t ni topamiz: $t=\sqrt{\frac{x-1}{7}}$, $t=\frac{y}{3}$. Bu ifodalardan $\frac{y}{3}=\sqrt{\frac{x-1}{7}}$ tenglamaga kelamiz. Bundan $y=3\sqrt{\frac{x-1}{7}}$ funksiyani topamiz. Demak, izlangan funksiya $y=3\sqrt{\frac{x-1}{7}}$ ekan. ▲

4- misol. $\begin{cases} x=4\sin t, \\ y=3\cos t \end{cases}$ parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziq bo‘ladi, bu yerda $0\leq t\leq 2\pi$?

△Berilgan tengliklardan $\sin t=\frac{x}{4}$ va $\cos t=\frac{y}{3}$ ekanini topamiz. $\sin^2 t+\cos^2 t=1$ ayniyatdan foydalanib, $(\frac{x}{4})^2+(\frac{y}{3})^2=1$ yoki $\frac{x^2}{16}+\frac{y^2}{9}=1$ tenglamani hosil qilamiz. Bu tenglama bilan berilgan nuqtalar to‘plami markazi koordinata boshida va yarim o‘qlari $a=4$, $b=3$ bo‘lgan *ellips* deb nomlanadi. ▲

Savol va topshiriqlar

Parametrik ko‘rinishda berilgan funksiyalarga misollar keltiring.

Mashqlar

173. Moddiy nuqtaning koordinatalari parametrik ko‘rinishda berilgan. Bu moddiy nuqta harakati davomida chizgan chiziqning (moddiy nuqta trayektoriyasining) formulasini toping. Mos rasm chizing:

1) $\begin{cases} x = 2t+1, \\ y = 4t+8; \end{cases}$ 2) $\begin{cases} x = 6t+4, \\ y = 9t+3; \end{cases}$ 3) $\begin{cases} x = 4t+9, \\ y = 7t+18; \end{cases}$ 4) $\begin{cases} x = 12t+11, \\ y = 15t+18. \end{cases}$

174. Moddiy nuqta koordinatalari parametrik ko‘rinishda berilgan. x va y koordinatalar orasidagi bog‘lanishni aniqlang:

1) $\begin{cases} x = 17t^2+1, \\ y = 13t; \end{cases}$ 2) $\begin{cases} x = 27t^2+21, \\ y = 23t; \end{cases}$ 3) $\begin{cases} x = 37t^2+31, \\ y = 33t; \end{cases}$ 4) $\begin{cases} x = 47t^2+41, \\ y = 43t. \end{cases}$

175. Parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziqdan iborat? Mos rasmni chizing:

1) $\begin{cases} x = 7\sin t, \\ y = 7\cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 2) $\begin{cases} x = \sin t, \\ y = \cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 3) $\begin{cases} x = 5\sin t, \\ y = 5\cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 4) $\begin{cases} x = 9\sin t, \\ y = 9\cos t, \\ 0 \le t \le 2\pi. \end{cases}$

176. Parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziqdan iborat? Mos rasmni chizing:

1) $\begin{cases} x = 6\sin t+3, \\ y = 6\cos t+7, \\ 0 \le t \le 2\pi; \end{cases}$ 2) $\begin{cases} x = 3\sin t, \\ y = 3\cos t-1, \\ 0 \le t \le 2\pi; \end{cases}$ 3) $\begin{cases} x = 2\sin t-3, \\ y = 2\cos t+7, \\ 0 \le t \le 2\pi. \end{cases}$

71 KO‘RSATKICHLI FUNKSIYA VA UNING GRAFIGI

Daraja va uning xossalari

Haqiqiy son ko‘rsatkichli daraja quyidagi xossalarga ega ($a>0$, $a\neq1$):

1) $a^x \cdot a^y = a^{x+y}$; 2) $a^x : a^y = a^{x-y}$; 3) $(a^x)^y = a^{x\cdot y}$;

4) $(a\cdot b)^x = a^x \cdot b^x$; 5) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$;

6) agar $0<a<b$ va $x>0$ bo‘lsa, $a^x<b^x$; 7) agar $0<a<b$ va $x<0$ bo‘lsa, $a^x>b^x$;
8) agar $x<y$ va $a>1$ bo‘lsa, $a^x<a^y$; 9) agar $x<y$ va $0<a<1$ bo‘lsa, $a^x>a^y$
bo‘ladi.

1- misol. Taqqoslang: $2^{-\sqrt{3}}$ va $3^{-\sqrt{3}}$.

△ 7- xossaga ko‘ra $0<2<3$ va $-\sqrt{3}<0$ bo‘lgani uchun $2^{-\sqrt{3}} > 3^{-\sqrt{3}}$. ▲

2- misol. Taqqoslang: $\left(\frac{1}{2}\right)^{0,2}$ va $\left(\frac{1}{2}\right)^{0,3}$.

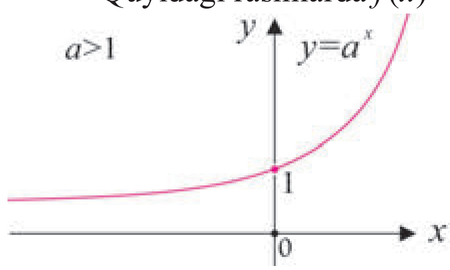
△ 9- xossaga ko‘ra 0,2<0,3 va $0<\frac{1}{2}<1$ bo‘lgani uchun $\left(\frac{1}{2}\right)^{0,2}>\left(\frac{1}{2}\right)^{0,3}$. ▲

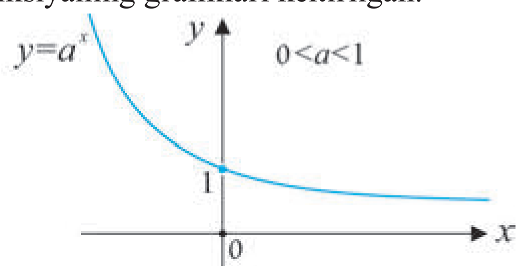
Ko‘rsatkichli funksiya va uning xossalari

$f(x)=a^x$, $a>0$, $a\neq 1$ ko‘rinishdagi funksiya *ko‘rsatkichli funksiya* deyiladi. Bunday funksiya quyidagi xossalarga ega:

1) aniqlanish sohasi $(-\infty;\ +\infty)$ oraliqdan iborat;
2) qiymatlar sohasi $(0;\ +\infty)$ oraliqdan iborat;
3) barcha $a(a>0,\ a\neq 1)$ uchun $a^0=1$;
4) $a>1$ bo‘lsa, funksiya o‘suvchi;
5) $0<a<1$ bo‘lsa, funksiya kamayuvchidir.

Quyidagi rasmlarda $f(x)=a^x$ funksiyaning grafiklari keltirilgan.





Savol va topshiriqlar

1. Haqiqiy son ko‘rsatkichli darajaning xossalarini ayting. Misollar keltiring.
2. Ko‘rsatkichli funksiyaning xossalarini ayting.

Mashqlar

177. Hisoblang:

1) $\left(\left(\sqrt{3}\right)^{\sqrt{2}}\right)^{\sqrt{2}}$; 2) $9^{\sqrt{3}}:3^{2\sqrt{3}}$ 3) $\left(2^{\sqrt[3]{4}}\right)^{\sqrt[3]{2}}$; 4) $4^{6\sqrt{2}-1}\cdot 16^{1-3\sqrt{2}}$.

Taqqoslang (**178–179**):

178. 1) $2^{-\sqrt{3}}$ va 1; 2) $4^{-\sqrt{6}}$ va $\left(\frac{1}{2}\right)^{4}$; 3) $\left(\frac{1}{3}\right)^{\sqrt{5}}$ va 1.

179. 1) $-3^{\sqrt{2}}$ va 1; 2) $\left(\frac{1}{2}\right)^{-\sqrt{2}}$ va $\left(\frac{1}{3}\right)^{-\sqrt{2}}$; 3) $\left(\frac{1}{2}\right)^{\sqrt{2}}$ va $\left(\frac{1}{3}\right)^{\sqrt{2}}$.

180. Funksiyalarning o‘suvchi yoki kamayuvchi ekanini aniqlang (**180–182**):

1) $y=4^x$; 2) $y=-3^x$; 3) $y=5^x-2$; 4) $y=-\left(\frac{1}{2}\right)^x+1$.

181. 1) $y=\sqrt{3}^x$; 2) $y=(\frac{1}{\sqrt{3}})^x$; 3) $y=(\frac{\pi}{3})^x$; 4) $y=(\sqrt{3}-1)^x$.

182. 1) $y=(\sqrt{3}-1)^{-x}$; 2) $y=(\sqrt{10}-2)^x$; 3) $y=\left(\pi-\sqrt{2}\right)^x-3$.

72-74 BEVOSITA YECHILADIGAN KO'RSATKICHLI TENGSIZLIKLAR

$a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq1$ ko'rinishdagi tengsizlik

$a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq1$ tengsizlik ko'rsatkichli tengsizlikka misol bo'la oladi. Bu tengsizlik $a>1$ bo'lganda $f(x)>g(x)$ tengsizlikka, $0<a<1$ bo'lganda esa $f(x)<g(x)$ tengsizlikka tengkuchlidir.

1- misol. Tengsizlikni yeching: $3^{x+5}>3^{2-5x}$.

△ $a=3>1$ bo'lgani uchun berilgan tengsizlik $x+5>2-5x$ tengsizlikka tengkuchli. Bundan $6x>-3$ yoki $x>-0,5$ ekanini topamiz. Demak, tengsizlikning yechimi $(-0,5;\ \infty)$ oraliqdan iborat. *Javob*: $x\in(-0,5;\ \infty)$. ▲

2- misol. Tengsizlikni yeching: $2\cdot3^{x+2}-2\cdot3^{x+1}-5\cdot3^x<63$.

△ 3^x ni qavsdan tashqariga chiqaramiz: $3^x(2\cdot3^2-2\cdot3-5\cdot1)<63$. Soddalashtirib, $3^x<9$ tengsizlikni hosil qilamiz. Bundan $x<2$. *Javob*: $x\in(-\infty;\ 2)$. ▲

3- misol. Tengsizlikni yeching: $8^{5x^2-46}\geq8^{2(x^2+1)}$.

△ $a=8>1$ bo'lgani uchun tengsizlik $5x^2-46\geq2(x^2+1)$ tengsizlikka tengkuchli. Shu tengsizlikni yechamiz: $3x^2\geq48$, bundan $x^2\geq16$. Demak, berilgan tengsizlikning yechimi $x\in(-\infty;\ -4]\cup[4;\ +\infty)$ bo'ladi. ▲

$a^x<b$ tengsizlikning ($a>0$, $a\neq1$) $b<0$ bo'lganda yechimi yo'q va $a^x>b$ tengsizlikning $b<0$ bo'lganda yechimi $(-\infty;+\infty)$ oraliqdan iborat ekanligi ravshan.

4- misol. Tengsizlikni yeching: $4^x+2^x-6\geq0$.

△ $2^x=t$ almashtirish kiritamiz, natijada $t^2+t-6\geq0$ kvadrat tengsizlik hosil bo'ladi. Bundan $t\leq-3$, $t\geq2$ ekanini topamiz va $2^x\geq2$ hamda $2^x\leq-3$ tengsizliklarga kelamiz. 1- tengsizlikdan $x\geq1$ yechim topiladi, 2- tengsizlikning esa yechimi yo'q. Demak, berilgan tengsizlikning yechimi $[1;+\infty;)$ oraliqdan iborat. *Javob*: $x\in[1;\ +\infty)$. ▲

Savol va topshiriqlar

$a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq1$ tengsizlik haqida ma'lumot bering. Misol keltiring.

Mashqlar

183. Tengsizlikni yeching (**183–184**):

1) $4^{3x+5}\le 4^{3-5x}$; 2) $7^{4x+5}<7^{9-5x}$; 3) $6^{x+5}>6^{3x}$, 4) $8^{x+5}\le 8^{2-5x}$;
5) $11^{x}<11^{2+5x}$; 6) $2^{x-5}>2^{25x}$; 7) $2\cdot 2^{x+2}-3\cdot 2^{x+1}-5\cdot 2^{x}\le -6$;
8) $3\cdot 5^{x+3}-5^{x+2}-2\cdot 5^{x+1}<68$; 9) $2\cdot 4^{x+2}+4^{x+1}-5\cdot 4^{x}\le 31$;
10) $2\cdot 7^{x+2}-2\cdot 7^{x+1}-14\cdot 7^{x}<10$. 11) $13^{x^2+46}\le 13^{x^2+25x}$;
12) $3^{x^2-4x}<3^{2(x^2-15)}$; 13) $7^{2x^2-4}\le 7^{3(x^2-x)}$.

184. 1) $9^{x}+3^{x}-6\le 84$; 2) $25^{x}+5^{x}-30>0$;
3) $5\cdot 4^{x}+2^{x}-6\le 0$; 4) $9^{x}+3^{x}-12>0$.

Nazorat ishi namunasi

1. $\begin{cases} x=7\sin 5t \\ y=7\cos 5t \end{cases}$ ko‘rinishidagi funksiya grafigini yasang.

2. $y=11^{x}+7$ funksiyaning xossalarini yozing.

Tengsizliklarni yeching (**3–5**):

3. $6^{x^2-7x-1}<6^{7}$. 4. $\left(\frac{1}{2}\right)^{17x}\ge\left(\frac{1}{2}\right)^{54-x}$. 5. $0,7^{-3x}\le 1$.

75-78 LOGARIFM HAQIDA TUSHUNCHA. LOGARIFMIK FUNKSIYA. ENG SODDA LOGARIFMIK TENGLAMA VA TENGSIZLIKLAR

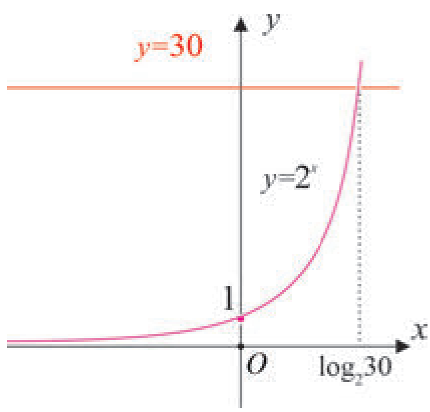
Logarifm haqida tushuncha

$2^{x}=32$ tenglamaning ildizi $x=5$, ammo $2^{x}=30$ tenglamaning ildizi qanday topiladi? Bu kabi tenglamalarni yechish uchun sonning logarifmi tushunchasi kiritiladi. $2^{x}=30$ *tenglama yagona ildizga ega*. Uni 32- rasmdan ko‘rish mumkin.

Bu ildiz 30 sonining 2 asosga ko‘ra logarifmi deyiladi va $\log_2 30$ kabi belgilanadi. Demak, $2^{x}=30$ tenglamaning ildizi $x=\log_2 30$ sondir.

Ushbu ta’rifni kiritamiz:

b musbat sonning *a* asosga ko‘ra logarifmi deb, *b* sonni hosil qilish uchun asos *a* ni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi va $\log_a b$ kabi belgilanadi. Asos $a>0$ va $a\ne 1$ shartni qanoatlantirishi kerak.



32- rasm.

Masalan, $\log_3 9=2$, chunki $9=3^2$. Shuningdek, $\log_2 \frac{1}{8}=-3$; $\log_5 5=1$; $\log_7 1=0$.

1- misol. Hisoblang: $\log_3 81$.

△ $3^4=81$ bo‘lgani uchun logarifmning ta’rifiga ko‘ra $\log_3 81=4$. ▲

Logarifmning xossalari

- asosiy logarifmik ayniyat: agar $a>0$, $a\neq 1$, $b>0$ bo‘lsa, $a^{\log_a b}=b$ tenglik o‘rinlidir;
- agar $a>0$, $a\neq 1$ bo‘lsa, $\log_a 1=0$; $\log_a a=1$;
- agar $a>0$, $a\neq 1$ va $x>0$, $y>0$ bo‘lsa, $\log_a(xy)=\log_a x+\log_a y$;
- agar $a>0$, $a\neq 1$ va $x>0$, $y>0$ bo‘lsa, $\log_a \frac{x}{y} = \log_a x-\log_a y$;
- agar $a>0$, $a\neq 1$, $x>0$ bo‘lsa $\log_a x^n = n\cdot\log_a x$;
- yangi asosga (bir asosdan boshqa asosga) o‘tish formulasi: agar $a>0$, $a\neq 1$, $x>0$, $b>0$, $b\neq 1$ bo‘lsa, $\log_a x=\frac{\log_b x}{\log_b a}$;
- agar $a>0$, $a\neq 1$, $b>0$, $b\neq 1$ bo‘lsa, $\log_a b\cdot\log_b a=1$.

$\log_{10} x=\lg x$ va $\log_e x=\ln x$ kabi belgilash qabul qilingan (e=2,718281...).

Bunda $\lg x$ ifoda x ning o‘nli logarifmi, $\ln x$ esa x ning natural logarifm deyiladi. $f(x)=\log_a x$ funksiya (bu yerda x – argument, $a>0$, $a\neq 1$) a asosli *logarifmik funksiya* deyiladi.

Logarifmik funksiyaning xossalari:

- aniqlanish sohasi $(0; +\infty)$ oraliq;
- qiymatlar sohasi $\mathbb{R}=(-\infty; +\infty)$;
- noli: $x=1$, ya’ni $\log_a 1=0$.
- $a>1$ bo‘lsa, logarifmik funksiya $(0; +\infty)$ oraliqda o‘suvchi;
- $0<a<1$ bo‘lsa, logarifmik funksiya $(0; +\infty)$ oraliqda kamayuvchi.

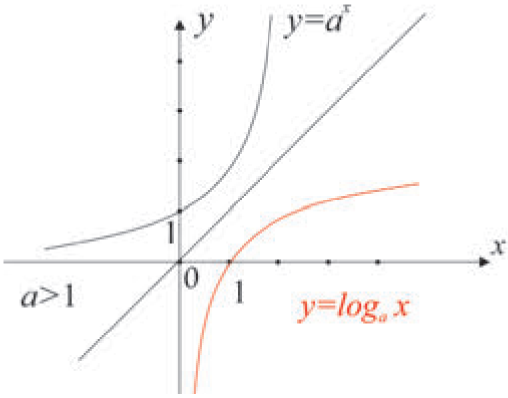
2- misol. Taqqoslang: $\log_{\frac{1}{2}} \frac{1}{3}$ va 0.

△ $\log_{\frac{1}{2}} 1=0$, asos $a=\frac{1}{2}$, ya’ni funksiya kamayuvchi $0<\frac{1}{2}<1$ va $0<\frac{1}{3}<1$ bo‘lganidan $\log_{\frac{1}{2}}\frac{1}{3}>\log_{\frac{1}{2}}1$ bo‘ladi. Demak, $\log_{\frac{1}{2}}\frac{1}{3}>0$ ekan. ▲

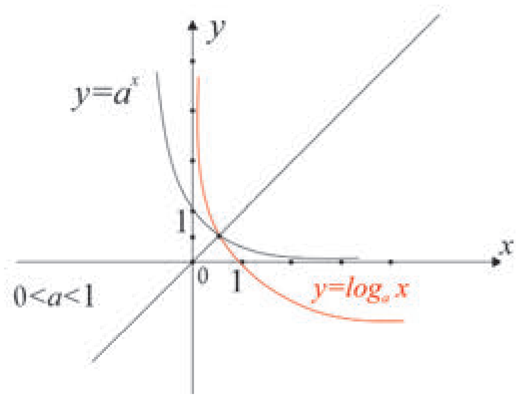
3- misol. Funksiyaning aniqlanish sohasini toping: $f(x)=\log_2 \frac{x^2-5x+6}{x-1}$.

△ Bu logarifmik funksiyaning aniqlanish sohasi x ning $\frac{x^2-5x+6}{x-1}>0$ tengsizlikni qanoatlantiruvchi barcha qiymatlari to‘plamidan iborat. Bu tengsizlikni yechib, funksiyaning aniqlanish sohasi $x \in (1;2) \cup (3;+\infty)$ ekanini topamiz. ▲

33 va 34- rasmlarda $y=a^x$ va $y=\log_a x$ funksiyalarning ($a>1$ va $0<a<1$ hollar uchun) grafiklari birgalikda tasvirlangan.



33- rasm.



34- rasm.

4- misol. Taqqoslang: $\log_3 2+\log_3 8$ va $\log_3(2+8)$.

△Logarifmning xossalaridan foydalanamiz: $\log_3 2+\log_3 8=\log_3(2\cdot 8)=\log_3 16$ $\log_3(2+8)=\log_3 10$. Logarifmning asosi $3>1$ bo‘lgani uchun $\log_3 16>\log_3 10$. Bundan: $\log_3 2+\log_3 8>\log_3(2+8)$.▲

5- misol. Hisoblang: $A=4^{\log_8 125}+27^{\frac{1}{3}-\frac{1}{2}\log_3 4}$.

△Logarifmning xossalaridan foydalanamiz: $\frac{1}{2}\log_3 4=\log_3 2;$

$\log_8 125=\dfrac{\log_2 125}{\log_2 8}=\dfrac{3\log_2 5}{3}=\log_2 5;\quad 4^{\log_8 125}=4^{\log_2 5}=2^{2\log_2 5}=2^{\log_2 25}=25.$

Shuningdek, $27^{\frac{1}{3}-\frac{1}{2}\log_3 4}=27^{\frac{1}{3}-\log_3 2}=27^{\frac{1}{3}}\cdot 27^{-\log_3 2}=$

$=3\cdot 3^{-3\log_3 2}=3\cdot 3^{\log_3\frac{1}{8}}=3\cdot\frac{1}{8}=\frac{3}{8}$. Demak, $A=25+\frac{3}{8}=25\frac{3}{8}$. ▲

6- misol. Hisoblang: $\dfrac{\lg 54+\lg\frac{1}{2}}{\lg 72-\lg 8}$.

△Logarifmning xossalaridan foydalanamiz:

$$\lg 54+\lg\frac{1}{2}=\lg(54\cdot\frac{1}{2})=\lg 27=\lg 3^3=3\lg 3,$$

$$\lg 72-\lg 8=\lg\frac{72}{8}=\lg 9=\lg 3^2=2\lg 3.$$

U holda: $\dfrac{\lg 54+\lg\frac{1}{2}}{\lg 72-\lg 8}=\dfrac{3\lg 3}{2\lg 3}=\dfrac{3}{2}$. *Javob*: $\dfrac{3}{2}$. ▲

Eng sodda logarifmik tenglama

$\log_a x=b$ ko‘rinishdagi tenglamani ($a>0$, $a\neq 1$, b – haqiqiy son) eng sodda logarifmik tenglama deyish mumkin. Tenglamaning yagona yechimi: $x=a^b$.

7- misol. Tenglamani yeching: $\log_3 x=\frac{1}{2}$.

△ Logarifm ta’rifiga ko‘ra, yechim $x=3^{\frac{1}{2}}=\sqrt{3}$. *Javob*: $x=\sqrt{3}$. ▲

8- misol. Tenglamani yeching: $\log_x 16=2$.

△ Logarifmning ta’rifiga ko‘ra, $x^2=16$ va $x>0$, $x\neq 1$ Demak, tenglamaning yechimi $x=4$ ekan. *Javob*: $x=4$. ▲

9- misol. Tenglamani yeching: $\log_2(x^2-5x+10)=4$.

△ Logarifmning ta’rifiga ko‘ra, $x^2-5x+10=2^4$ tenglamani hosil qilamiz. Kvadrat tenglamani yechib $x_1=-1$, $x_2=6$ ildizlarni topamiz. Demak, tenglamaning yechimi $\{-1; 6\}$ ekan. *Javob*: $x=-1$, $x=6$. ▲

10- misol. Tenglamani yeching: $\lg(2x-3)=\lg(x-1)$.

△ Logarifmning ta’rifiga ko‘ra, $2x–3>0$, $x>1$ bo‘lishi kerak. Tenglamaning aniqlanish sohasi $x>\frac{3}{2}$ oraliqdan iborat. Logarifmning xossasiga ko‘ra, $2x-3=x-1$ tenglamaga kelamiz, bundan $x=2$. Bu ildiz esa aniqlanish sohasiga tegishli. *Javob*: $x=2$. ▲

11- misol. Tenglamani yeching: $\log_x(x+2)=2$.

△ Tenglamaning aniqlanish sohasini topamiz: $x+2>0$ $x>0$, $x\neq 1$, ya’ni tenglama $(0,1)\cup(1;\infty)$ to‘plamda aniqlangan. Logarifmning ta’rifiga ko‘ra, $x+2=x^2$ tenglamani hosil qilamiz. Bu kvadrat tenglamani yechib $x_1=-1$, $x_2=2$ ildizlarni topamiz. Bu ildizlardan faqat $x=2$ aniqlanish sohasiga tegishli. Shuning uchun ham u berilgan tenglamaning yechimi bo‘ladi. *Javob*: $x=2$. ▲

12- misol. Tenglamani yeching: $\log_3^2 x-5\log_3 x+6=0$.

△ $t=\log_3 x$ belgilash kiritib, $t^2–5t+6=0$ kvadrat tenglamani hosil qilamiz. Uni yechib, $t=2$ va $t=3$ ildizlarni topamiz. Topilgan ildizlarni $t=\log_3 x$ ga qo‘yib, $\log_3 x=2$ va $\log_3 x=3$ tengliklarni olamiz. Bu tenglamalarning yechimlari, mos ravishda, 9 va 27 bo‘ladi. *Javob*: $x=9$, $x=27$. ▲

Eng sodda logarifmik tengsizlik

$\log_a x > b$ ko‘rinishdagi tengsizlikni ($a>0,\ a\neq 1,\ b$ – haqiqiy son) eng sodda logarifmik tengsizlik deyish mumkin.

13- misol. Tengsizlikni yeching: $\log_{\frac{1}{2}}(3-x) > -3$.

△ $3-x>0$ bo‘lishi kerak, $-3=\log_{\frac{1}{2}}8$ ekanidan $\log_{\frac{1}{2}}(3-x)>\log_{\frac{1}{2}}8$. Asos $a=\frac{1}{2}<1$ bo‘lgani uchun logarifmik funksiya kamayuvchi, demak, $3-x<8$ va $0<3-x<8$. Bundan $-3<-x<5$ yoki $-5<x<3$ tengsizliklarga kelamiz.
Javob: $x\in(-5;\ 3)$. ▲

14- misol. Tengsizlikni yeching: $\lg(x+1)<\lg(2x-3)$.
△ Logarifmik funksiyaning xossalaridan quyidagi tengsizliklar sistemasini olamiz:

$$\begin{cases} x+1<2x-3, \\ x+1>0, \\ 2x-3>0 \end{cases} \quad \text{yoki} \quad \begin{cases} x>4, \\ x>-1, \\ x>\dfrac{3}{2}. \end{cases}$$

Bu sistemaning yechimi $(4;\ +\infty)$ oraliqdan iborat. *Javob*: $x\in(4;\ +\infty)$. ▲

15- misol. Tengsizlikni yeching: $\log^2_{\frac{1}{2}} x-9\leq 0$.

△ Logarifmik funksiya ta’rifiga ko‘ra, $x>0$ bo‘lishi kerak. $t=\log_{\frac{1}{2}}x$ belgilash kiritamiz. U holda $t^2-9\leq 0$ tengsizlikni hosil qilamiz. Buni yechib $-3\leq t\leq 3$, ya’ni $-3\leq\log_{\frac{1}{2}}x\leq 3$ tengsizliklarga kelamiz. $-3=\log_{\frac{1}{2}}8;\ 3=\log_{\frac{1}{2}}\frac{1}{8}$ ekanidan $\log_{\frac{1}{2}}8\leq\log_{\frac{1}{2}}x\leq\log_{\frac{1}{2}}\frac{1}{8}$. Asos $a=\frac{1}{2}<1$ bo‘lgani uchun $y=\log_{\frac{1}{2}}x$ funksiya kamayuvchi, demak, $\frac{1}{8}\leq x\leq 8$ bo‘lishi kerak. *Javob*: $x\in[\frac{1}{8};8]$. ▲

Savol va topshiriqlar

1. Logarifmga ta’rif bering. Misol keltiring.
2. Logarifmning xossalarini ayting. Misolda tushuntiring.
3. Logarifmik funksiyalarning xossalarini ayting.
4. Eng sodda logarifmik tenglama nima va u qanday yechiladi?

5. Eng sodda logarifmik tengsizlik nima va u qanday yechiladi? Misol keltiring.

Mashqlar

185. Hisoblang:

1) $\log_5 125$; 2) $\log_{\frac{1}{3}} 9$; 3) $\log_5 0,04$; 4) $\log_{0,1} 1000$; 5) $\log_3 \frac{1}{27}$.

186. Taqqoslang:

1) $\log_2 3$ va $\log_2 5$; 2) $\frac{\log_2 3}{\log_2 5}$ va $\log_5 4$; 3) $\log_{\frac{1}{2}} 3$ va $\log_{\frac{1}{2}} 5$;

4) $\log_2 3$ va 1; 5) $\log_3 2 + \log_3 5$ va $\log_3(2+5)$; 6) $\log_7 \frac{1}{2}$ va 0.

187. Hisoblang:

1) $1,5^{\log_{1,5} 2}$; 2) $e^{\ln 5}$; 3) $2^{3\log_2 5}$; 4) $3^{2+\log_3 5}$; 5) $7^{-2\log_7 6}$;

6) $3^{3-\log_3 54}$; 7) $\log_6 2 + \log_6 18$; 8) $\lg 25 + \lg 4$; 9) $\log_3 \frac{5}{9} + \log_3 \frac{1}{5}$;

10) $\frac{\lg 2 + \lg 162}{2\lg 3 + \lg 2}$; 11) $\log_4 7 - \log_4 \frac{7}{16}$; 12) $\frac{\ln 64}{\ln 4}$.

188. Funksiyalarning aniqlanish sohasini toping:

1) $y = \log_3(2x-5)$; 2) $y = \log_7(x^2 - 2x - 3)$; 3) $y = \log_5(4 - x^2)$.

4) $y = \log_2(x^2 - 2x + 1)$; 5) $y = \log_{\sqrt{2}}(3 - x)$; 6) $y = \log_2 \frac{x-1}{x+2}$.

189. Funksiyaning grafigini chizing:

1) $y = \log_2 x$; 2) $y = \log_{\frac{1}{3}} x$; 3) $y = \log_4(x-1)$; 4) $y = -\log_3 x$.

190. Tenglamani yeching:

1) $\log_2 x = -5$; 2) $\log_{\sqrt{3}} x = 0$; 3) $\log_{\frac{1}{2}} x = -2$; 4) $\log_x 128 = 7$;

5) $\log_9 x = \frac{1}{2}$; 6) $\log_{\sqrt{x}} 27 = 3$; 7) $\log_3 x = 5$;

8) $\log_2(x-5) = \log_2(4x+1)$; 9) $\log_{\frac{1}{2}} x = -2$; 10) $\log_5(3-2x) = \log_{\frac{1}{5}} x$;

11) $\log_{\frac{1}{3}}(3x-6) = -2$; 12) $\log_2(x+1) + \log_2(8-x) = 3$; 13) $\log_x 5 = 2$;

14) $\lg(x^2+x-10)-\lg(x-3)=1$; | 15) $\log_7^2 x - \log_7 x = 2$;

16) $5^{4-x}=6$; | 17) $\log_x 3+\log_3 x=2$; | 18) $5^{x^2}=6$; | 19) $5^{x^2}=\frac{1}{2}$;

20) $\lg(x^2-6x+19)=1$; | 21) $\log_5(5^x-4)=1-x$; | 22) $\lg(x^2-21)=2$.

191. Tengsizlikni yeching:

1) $\log_8 x>2$; | 2) $\log_3^2 x-3>2\cdot\log_3 x$; | 3) $\log_8 x<2$; | 4) $\log_{\frac{1}{2}} x>1$;

5) $\lg(3-2x)>1$; | 6) $2^{x+1}<3$; | 7) $\log_3(2x-4)<\log_3(x+1)$; | 8) $2^{|x+1|}>3$.

79-81 KO'RSATKICHLI VA LOGARIFMIK FUNKSIYALAR YORDAMIDA MODELLASHTIRISH

1-misol. Bakteriya ma'lum vaqtdan (bir necha soat yoki minutlardan) so'ng ikkiga bo'linadi va bakteriyalar soni ikki karra ortadi. Navbatdagi vaqtdan so'ng mazkur ikkita bakteriya ham ikkiga bo'linadi va populatsiya miqdori (bakteriyalar umumiy soni) yana ikki karra ortadi; endi, bakteriyalar soni to'rtta bo'ldi. Bu ko'payish jarayoni qulay sharoitlarda (populatsiya uchun zarur resurslar: joy, oziqa, suv, energiya va hokazolar mavjud bo'lganda) davom etaveradi.

Faraz qilaylik, dastlab 10 millionta bakteriya borligi, bunday bakteriyalar bir soatdan so'ng ikkiga bo'linishi ma'lum bo'lsin. Quyidagi jadval $t = 1, 2, 3, 4$ soat o'tganda b populatsiya miqdori qanday o'zgarishini ifodalaydi:

t (soat)	0	1	2	3	4
b_t (million)	10	20	40	80	160

Shu bilan birga, barcha bakteriyalar ham har soatda bir vaqtda sinxron ravishda ikkiga bo'linmasligini ma'lum. Bunday holatda t butun son bo'lmaganda (masalan, $t=1\frac{1}{2}$ soat) bakteriyalar populatsiyasi miqdorini topish masalasi turibdi.

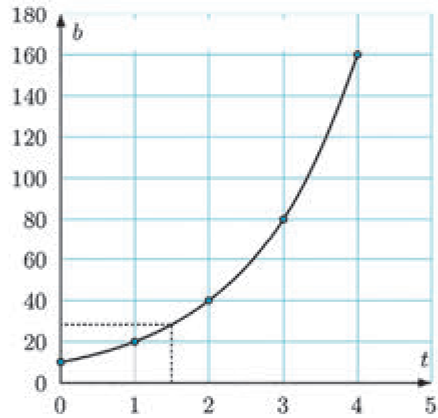
a) $b_1, b_2, \ldots$ ketma-ketlik qanday ketma-ketlik?

b) Tekislikdagi to'g'ri burchakli koordinatalar sistemasida jadval bo'yicha mos nuqtalarni belgilab, so'ng hosil bo'lgan nuqtalarni silliq chiziq bilan tutashtiring.

c) $t=1\frac{1}{2}$ soat o'tgandan keyin bakteriyalar populatsiyasi qanday bo'ladi?

d) Bakteriyalar populatsiyasining ixtiyoriy t vaqtga nisbatan o'zgarishini qanday funksiya yordamida modellashtirsa bo'ladi?

△ Jadvalning 2- qatordagi $b_1, b_2, \ldots$ sonlar ketma-ketligi maxraji 2 ga teng bo'lgan geometrik progressiya ekanligi ravshan. Uning umumiy ko'rinishi quyidagicha bo'ladi: $b_t=20\cdot 2^{t-1}$, bu yerda $t = 1, 2, 3, 4$.



Tekislikdagi koordinatalar sistemasida jadval bo'yicha mos nuqtalarni belgilab, so'ng hosil bo'lgan nuqtalarni silliq chiziq bilan tutashtiraylik:

$t=1\frac{1}{2}$ soat o'tganda bakteriyalar populatsiyasi taqriban 28 million ekanligini ko'rsak bo'ladi.

Hosil bo'lgan egri chiziq shakli ko'rsatkichli funksiya grafigiga o'xshashligi ko'rinib turibdi. Bu funksiyani $b(t)$ deb belgilab (bu yerda $t \geq 0$), yoza olamiz: $b(t)=20\cdot 2^{t-1}=10\cdot 2^t$.▲

Umumiy holda, $b(t)=b_0 a^t$ qonuniyat bilan o'zgaradigan miqdor (bu yerda $b_0>0,\ a>1,\ t\geq 0$) *eksponentsial o'suvchi* miqdor deyiladi.

Quyidagi xulosaga ega bo'lamiz:

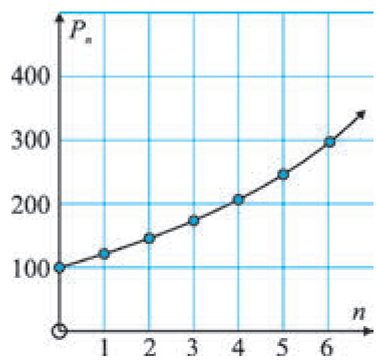
Agar populatsiyaning miqdoriy o'sishi uning boshlang'ich (dastlabki) soniga proporsional bo'lsa, bunday populatsiya eksponensial ko'payadi.

"Eksponensial o'sish" iborasi odatda qandaydir shiddatli, to'xtovsiz o'sish jarayonini ifodalaydi. Masalan, jonzotlar populatsiyasi, biror mamlakat aholisining shiddatli o'sishini matbuotda shunday ta'riflashadi.

2- misol. Epidemiologiya xizmatining ma'lumotiga ko'ra, sichqonlar populatsiyasi miqdori qulay sharoitlarda har haftada 20% ortar ekan. Dastlab 100 ta sichqon bo'lsa, ularning populatsiyasi miqdori qanday qonuniyat bilan o'sishini toping.

△Agar P_n deb n hafta davomidagi populatsiya miqdorini belgilasak, quyidagilarga ega bo'lamiz: $P_0=100$ (dastlabki miqdor), $P_1=P_0\cdot 1{,}2=100\cdot 1{,}2$, $P_2=P_1\cdot 1{,}2=100\cdot(1{,}2)^2$, $P_3=P_2\cdot 1{,}2=100\cdot(1{,}2)^3$, va h.k. n hafta davomidagi populatsiya miqdori $P_n=100\cdot(1{,}2)^n$ bo'ladi.▲

Kalkulatordan foydalanib, mos qiymatlarni hisoblasak, quyidagi grafikka ega bo'lamiz:



Ko'rinib turibdiki, 6 haftada populatsiya miqdori qariyb 3 marta ortar ekan.

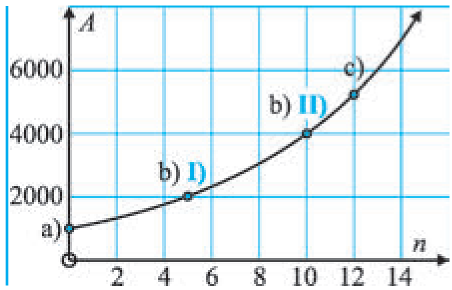
3- misol. Entomolog olim chigirtkalar populatsiyasining qishloq xo'jaligi dalalariga zarar yetkazishini o‘rganganda zarar ko‘rgan maydonlar yuzi $A_n=1000\cdot 2^{0,2n}$ (gektar) qonuniyat bilan o‘zgarishini aniqladi, bu yerda n haftalar soni.

a) Dastlab qanday maydonga zarar yetkazilgan?

b) **I)** 5; **II)** 10 haftada qanday maydonga zarar yetkaziladi?

c) Kalkulatordan foydalanib, 12 haftada qanday maydonga zarar yetkazilishini toping.

d) Zarar ko‘rgan maydon yuzi bilan haftalar soni orasidagi bog‘lanish qonuniyatining grafigini chizing.



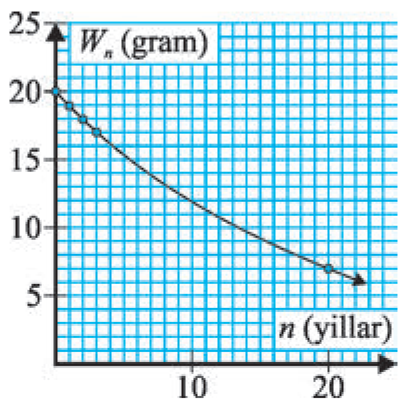
△ a) $A_0=1000\cdot 2^{0,2\cdot 0}=1000$ (gektar). Demak, dastlab 1000 ga maydonga zarar yetkazilgan.

b) **I)** $A_3=1000\cdot 2^{0,2\cdot 5}=2000$ zarar ko‘rgan maydon yuzi 2000 (ga) ga teng.

II) $A_{10}=1000\cdot 2^{0,2\cdot 10}=4000$ zarar ko‘rgan maydon yuzi 4000 (ga) ga teng.

c) $A_{12}=1000\cdot 2^{0,2\cdot 12}=1000\cdot 2^{2,4}\approx 5280$ zarar ko‘rgan maydon yuzi taqriban 5280 gektarga teng. ▲

4- misol. Radioaktiv yemirilish natijasida massasi 20 gramm bo‘lgan radioaktiv modda har yili 5% ga kamayadi. W_n deb moddaning n yildagi massasini belgilasak,



$W_0=20$ g;

$W_1=W_0\cdot 0{,}95=20\cdot 0{,}95$ g;

$W_2=W_1\cdot 0{,}95=20\cdot (0{,}95)^2$ g;

$W_3=W_2\cdot 0{,}95=20\cdot (0{,}95)^3$ g;

$W_{20}=20\cdot (0{,}95)^{20}\approx 7{,}2$ g;

$W_{100}=20\cdot (0{,}95)^{100}\approx 0{,}1$ g

tengliklarga ega bo‘lamiz.

Bundan $W_n=20\cdot (0{,}95)^n$.

$b(t)=b_0a^t$ qonuniyat bilan o‘zgaradigan miqdor (bu yerda $b_0>0$, $0<a<1$, $t\geq 0$) *eksponentsial kamayuvchi* miqdor deyiladi.

5- misol. Iste’mol qilingan dori inson tanasiga asta-sekin singib, uning t soatdan so‘ng qolayotgan miqdori (dozasi) $D(t)=120\cdot (0{,}9)^t$ (mg) qonuniyat bilan o‘zgaradi.

a) $t=0$, 4, 12, 24 soat bo‘lganda $D(t)$ ni toping.

b) Dastlab inson tanasiga qanday doza kiritilgan?

c) *a)* dagi ma'lumotlardan foydalanib, $D(t)$ grafigini tasvirlang, bu yerda $t\geq 0$.

d) Grafikdan foydalanib, 25 mg miqdordagi dori inson tanasida qancha vaqt qolishini baholang.

△ a) $D(t)=120\cdot(0,9)^t$ mg

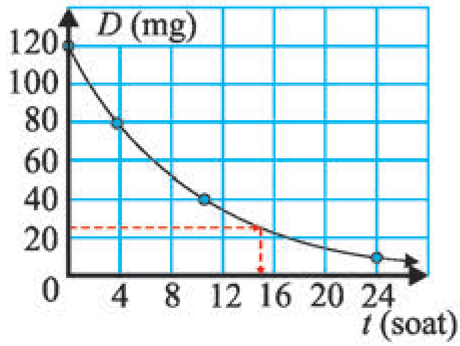
$D(0)=120\cdot(0,9)^0=120$ mg;

$D(4)=120\cdot(0,9)^4\approx 78,7$ mg;

$D(12)=120\cdot(0,9)^{12}\approx 33,9$ mg;

$D(24)=120\cdot(0,9)^{24}\approx 9,57$ mg;

b) $D(0)=120$ bo'lgani uchun dastlab 120 (mg) dori kiritilgan.

c) 

Shu grafikdan foydalanib, inson tanasiga kiritilgan 120mg dorining taxminan 15 soatdan so'ng 25 mgi qolishini aniqlaymiz. ▲

6- misol. Radioaktiv yemirilish natijasida radioaktiv modda massasi $W_t=W_0\cdot 2^{-0,001t}$ (g) qonuniyat bo'yicha o'zgaradi, bu yerda t- yillar.

a) Dastlab modda qanday massaga ega bo'lgan?

b) 200 yildan so'ng moddaning necha foizi qoladi?

△ $t=0$ bo'lganda $W_t=W_0\cdot 2^0=W_0$ bo'ladi. Demak, moddaning dastlabki massasi W_0 ekan. $t=200$ bo'lganda $W_{200}=W_0\cdot 2^{-0,001\cdot 200}=W_0\cdot 2^{-0,2}\approx W_0\cdot 0,8706$. Demak, 200 yildan so'ng moddaning taxminan 87,1 foizi qoladi. ▲

7- misol. Dengiz sathidan h km balandlikka ko'tarilganimizda, atmosfera bosimi $p=76\cdot 2,7^{-\frac{h}{8}}$ (sm simob ustuni) qonuniyat bilan o'zgarar ekan. 5,6 km balandlikda atmosfera bosimi qanday bo'ladi?

8- misol. Dengiz sathidan balandlik $h=\dfrac{8000}{0,4343}\lg\dfrac{p_0}{p}$ formula bilan hisoblanadi, bu yerda p_0=760 mm simob ustuni – dengiz sathidagi atmosfera bosimi, p esa h (m) balandlikdagi atmosfera bosimi. Alpinistlar toqqa ko'tarilganda bosim 304 mm simob ustuni bo'lganini aniqlashdi. Alpinistlar qanday balandlikka ko'tarilishdi?

$$h=\frac{8000}{0,4343}\lg\frac{760}{304}\approx 7330,2 \text{ m}.$$

9- misol. Radioaktiv modda massasi vaqt o‘tish bilan $m(t)=m_0\cdot\left(\frac{1}{2}\right)^{\frac{t}{T}}$ qonuniyatga ko‘ra kamayadi, bu yerda m_0 – boshlang‘ich vaqtdagi massa, m esa t vaqtdagi massa, T – radioaktiv yemirilish tezligini ifodalovchi koeffitsiyent (yarim yemirilish davri).

Moddaning hozirgi kunda saqlanib qolgan m massasini bilsak, necha yilda massa m_0 dan m gacha kamayganini topa olamiz:

$$t=-T\log_2\left(\frac{m(t)}{m_0}\right).$$

Bunday munosabat tarixiy tadqiqotlarda ham qo‘llanilishini aytish joiz.

10- misol. Tabiiy til lug‘atidagi so‘zlar soni vaqt o‘tishi bilan $N(t)=N_0\cdot e^{-\lambda t}$ qonuniyat bilan kamayishi kuzatilgan, bu yerda N_0 – boshlang‘ich vaqtdagi so‘zlar soni, $N(t)$ – t (ming yillar) vaqtdagi saqlanib qolgan so‘zlar soni, λ – tildagi so‘zlarning saqlanib qolishini ifodalovchi koeffitsiyent.

Hozirgi kunda saqlanib qolgan so‘zlar $N(t)$ miqdorini bilsak, necha yilda so‘zlar hajmi N_0 dan $N(t)$ gacha kamayganini topa olamiz:

$$t=-\frac{1}{\lambda}\cdot\ln\left(\frac{N(t)}{N_0}\right).$$

11- masala. Dastlab shahar aholisi a kishi bo‘lib, aholi soni har yili 10 % ga ortsa, aholining x yildan keyin qancha bo‘lishini aniqlovchi formulani toping.

△ Murakkab foiz formulasiga ko‘ra, shahar aholisi soni x yildan so‘ng $y=a\cdot\left(\frac{100+10}{100}\right)^x=a\cdot(1{,}1)^x$ bo‘ladi: Demak, $y=a\cdot(1{,}1)^x$ formula yordamida a berilganda x yildan so‘ng aholi soni qancha bo‘lishini aniqlash mumkin bo‘ladi. $a=1000000$ va yillar soni x bo‘yicha aholi sonini aniqlovchi jadvalni keltiramiz:

x	y	x	y
1	1 100 000	11	2 853 117
2	1 210 000	12	3 138 428
3	1 331 000	13	3 452 271
4	1 464 100	14	3 797 498
5	1 610 510	15	4 177 248
6	1 771 561	16	4 594 973
7	1 948 717	17	5 054 470
8	2 143 589	18	5 559 917
9	2 357 948	19	6 115 909
10	2 593 742	20	6 727 500

Jadvalga ko‘ra aholi soni 5 yildan so‘ng 1 610 510 kishi; 10 yildan so‘ng 2 593 742 kishi; 20 yildan so‘ng esa 6 727 500 kishi bo‘lar ekan.▲

12- masala. Dastlab shahar aholisi a kishi bo‘lib, aholi soni har yili 2 % ga kamaysa, aholining x yildan keyin qancha bo‘lishini aniqlovchi formulani toping.
△Murakkab foiz formulasiga ko‘ra, shahar aholisi soni x yildan so‘ng $y = a\cdot\left(\frac{100-2}{100}\right)^x = a\cdot 0{,}98^x$ bo‘ladi. Demak, $y=a\cdot 0{,}98^x$ formula yordamida a berilganda x yildan so‘ng aholi sonini aniqlash mumkin. a=2000000 va yillar soni x bo‘yicha aholi sonini aniqlovchi jadvalni keltiramiz:
Jadvalga ko‘ra aholi soni 5 yildan so‘ng 1 807 842 kishi; 10 yildan so‘ng 1 634 146 kishi; 20 yildan so‘ng esa 1 335 216 kishi bo‘lar ekan.▲

x	y	x	y
1	1 960 000	11	1 601 463
2	1 920 800	12	1 569 433
3	1 882 384	13	1 538 045
4	1 844 736	14	1 507 284
5	1 807 842	15	1 477 138
6	1 771 685	16	1 447 595
7	1 736 251	17	1 418 644
8	1 701 526	18	1 390 271
9	1 667 496	19	1 362 465
10	1 634 146	20	1 335 216

13- masala. Shahar aholisi dastlab a kishi edi. Agar aholi soni har yili 10 % ga ortsa, aholining x yildan keyin qancha bo‘lishini va necha yildan keyin k marta ortishini aniqlovchi formulani toping.
△Ma’lumki, $y=a\cdot 1{,}1^x$ va masala shartidan $y=k\cdot a$ ekanini hisobga olib, $k=1{,}1^x$ yoki $x=\log_{1,1}k$ formula topiladi. Quyida aholi soni k marta ortishi uchun kerakli yillar sonini aniqlovchi jadval keltirilgan:

k	y	k	y	k	y
1	0	6	19	11	25
2	7	7	20	12	26
3	12	8	22	13	27
4	15	9	23	14	28
5	17	10	24	15	28

Jadvaldan ma'lumki, aholi soni 2 marta ortishi uchun 7 yil;
5 marta ortishi uchun 17 yil;
10 marta ortishi uchun 24 yil kerak ekan. ▲

14- masala. Shahar aholisi har yili 2 % ga kamaysa hamda aholining boshlang'ich soni *a* nafar bo'lsa, aholining *x* yildan keyin qancha bo'lishini va necha yildan keyin *k* marta kamayishini aniqlovchi formulani toping.

△ Ma'lumki, $y=a\cdot 0{,}98^x$ va masala shartidan $y=\frac{a}{k}$ bo'lishini inobatga olib $1/k=0{,}98^x$ yoki $x=\log_{0,98}(1/k)$ formula topiladi. Quyida aholi soni *k* marta kamayishi uchun kerakli yillar sonini aniqlovchi jadval keltirilgan:

k	1/*k*	*x*	*k*	1/*k*	*x*
1	1	0	11	0,090909	119
2	0,5	34	12	0,083333	123
3	0,333333	54	13	0,076923	127
4	0,25	69	14	0,071429	131
5	0,2	80	15	0,066667	134
6	0,166667	89	16	0,0625	137
7	0,142857	96	17	0,058824	140
8	0,125	103	18	0,055556	143
9	0,111111	109	19	0,052632	146
10	0,1	114	20	0,05	148

Jadvaldan ma'lumki, aholi soni: 2 marta kamayishi uchun 34 yil;
5 marta kamayishi uchun 80 yil;
10 marta kamayishi uchun 114 yil kerak ekan. ▲

15- masala. 1935- yili amerikalik seysmolog Ch. Rixter zilzilalarni tasniflash uchun 1 – 9,5 ballik magnitudalar shkalasini taklif qilgan. Bunda zilzila vaqtida yuzaga keluvchi seysmik to'lqin energiyasi *intensivlik* deb nomlanuvchi kattalik orqali baholandi. Rixter shkalasida *intensivligi I bo'lgan zilzilaning R magnitudasi* $R=\lg I$ formula yordamida topilar ekan.

1966- yili Toshkentda 5,2 magnitudali, 2010- yili Gaitida esa 7 magnitudali zilzila ro'y bergan. Shu zilzilalarni intensivlik bo'yicha solishtiraylik.

△ Gaiti zilzilasi: $7=\lg I_1$, bundan $I_1=10^7=10\,000\,000$;
Toshkent zilzilasi: $5{,}2=\lg I_2$, bundan $I_2=10^{5,2}\approx 158\,489{,}3$.

Bundan $\frac{I_1}{I_2}\approx 63{,}1$. Demak, Gaitida Toshkentdagiga nisbatan taxminan 63 marta kuchliroq zilzila ro'y bergan. ▲

Savol va topshiriqlar

1. Ko‘rsatkichli modelga misol keltiring.
2. Logarifmik modelga misol keltiring.

Mashqlar

192. Tomorqaga ishlov berilmasa, t kundan so‘ng begona o‘tlar yuzi $A(t)=3\cdot2^{0,1t}$ (kv. m) bo‘lgan yer maydonini qoplab, foydali o‘simliklarga ziyon yetkazadi.

a) Dastlab qancha maydonga ziyon yetkazilgan?

b) **I)** 2; **II)** 10; **III)** 30 kunda qanday maydonga ziyon yetkaziladi?

c) *a), b)* da olingan ma’lumotlardan foydalanib, ziyon ko‘rgan maydon yuzining kunlar soniga bog‘lanish qonuniyati grafigini chizing.

193. Orolbo‘yi ekologik tizimini tiklash maqsadida noyob hayvonlar populatsiyasini ko‘paytirish loyihasida ekologlar 25 ta juftlik hayvonlarni ko‘paytirmoqchi. Tadqiqotlarga ko‘ra, berilgan sharoitlarda bu hayvonlar populatsiyasi miqdori $P_n=P_0\cdot1{,}23^n$ qonuniyat bilan o‘zgaradi, bu yerda P_n – n yildagi hayvonlar soni.

a) P_0 soni nimani bildiradi?

b) **I)** 2; **II)** 5; **III)** 10 yilda qanday populatsiyaga ega bo‘lamiz?

c) *a), b)* da olingan ma’lumotlardan foydalanib, populatsiya miqdorining yillar soniga bog‘lanish qonuniyati grafigini chizing.

194. Kimyoviy reaktsiya tezligi $V_t=V_0\cdot2^{0,05t}$ qonuniyat bilan o‘zgarar ekan, bu yerda t(°C) – temperatura.

a) 0 °C; b) 20 °C temperaturada reaktsiya tezligi qanday bo‘ladi?

c) 20 °C temperaturadagi reaktsiya tezligi 0 °C temperaturadagi reaksiya tezligiga nisbatan necha foiz ortadi?

d) $\left(\dfrac{V_{50}-V_{20}}{V_{20}}\right)\cdot100\%$ qiymatni hisoblang va ma’nosini tushuntiring.

195. 2017- yili Alyaska yarimoroli yonidagi orolga ayiqlarning 6 ta juftligi qo‘yib yuborildi. Dastlab orolda ayiqlar yo‘q edi. Ayiqlar populatsiyasi $B_t=B_0\cdot2^{0,18t}$ qonuniyat (bu yerda t – yillar) bilan o‘zgarsa, hisoblash vositalardan foydalanib, quyidagilarga javob bering:

a) B_0 soni nimani bildiradi? U nechaga teng?

b) 2037- yilda qanday populatsiyaga ega bo‘lamiz?

c) 2037- yildagi ayiqlar soni 2027- yildagi ayiqlar soniga nisbatan necha foiz ortadi?

196. Radioaktiv yemirilish natijasida radioaktiv modda massasi $W(t)=250\cdot(0{,}998)^t$ (g) qonuniyat bo‘yicha o‘zgaradi, bu yerda t – yillar.
a) Dastlab modda qanday massaga ega bo‘lgan?
b) **I)** 400; **II)** 800; **III)** 1200 yilda moddaning necha grammi qoladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.
d) Grafikdan foydalanib, modda qachon 125 mg miqdorda qolishini baholang.

197. Qaynoq suv sovitilganda uning T temperaturasi $T(t)=100\cdot2^{-0{,}02t}$ °C qonuniyat bilan o‘zgarar ekan, bu yerda t – minutlar.
a) Dastlab qanday temperatura bo‘lgan?
b) **I)** 15; **II)** 20 minutdan keyin temperatura nechaga teng bo‘ladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.
d) Grafikdan foydalanib, 78 minutdan keyin temperatura nechaga teng bo‘lishini baholang.

198. Elektr zanjirdagi tok kuchi $I_t=0{,}6\cdot2^{-5t}$ (A) qonuniyat bilan o‘zgarar ekan, bu yerda t – sekundlar.
a) Dastlab qanday tok kuchi bo‘lgan?
b) **I)** 0,1; **II)** 0,5; **III)** 1 sekunddan keyin tok kuchi nechaga teng bo‘ladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.

199. Dengizda d metr chuqurlikka nisbatan yorug‘lik $L(d)=L_0\cdot(0{,}9954)^d$ (kandela) qonuniyat bilan o‘zgarar ekan.
a) Dengiz tubida qanday yorug‘lik bo‘lgan?
b) 1000 metr chuqurlikdagi yorug‘lik necha foizga kamayadi?

200. 8 ta bakteriya populatsiyasi 2 soatdan so‘ng 100 tagacha o‘sdi. Shu sharoitlarda qachon populatsiya 500 taga yetadi?

201. Uyali aloqa kompaniyasi ma’lumotlariga ko‘ra, kompaniya uyali aloqasidan foydalanuvchilar soni $N(t)=100000e^{0{,}09t}$ formula yordamida ifodalanar ekan, bu yerda t – oylar. Hozirgi kunda 3 mln foydalanuvchilar borligi ma’lum bo‘lsa, kompaniya qachon ish boshlagan?

202. Ovqat mikroto‘lqinli pechdan olinganda, u $T(t)=80e^{-0{,}12t}$ qonuniyatga asosan soviydi, bu yerda t – minutlar. Hozir xona temperaturasi 22°C bo‘lsa, necha minutdan so‘ng ovqat shu temperaturagacha soviydi?

203. Sun’iy yo‘ldosh balandligi t (yillar) vaqt o‘tishi bilan $H(t)=30000e^{-0{,}2t}$ qonuniyat bilan o‘zgarar ekan.
a) 2 yildan so‘ng balandlik qanday bo‘lishini hisoblang.
b) Yo‘ldosh 320 km balandlikda bo‘lsa, u atmosferaning yuqori qatlamlarida yonib ketadi. Shu paytgacha qancha vaqt o‘tadi?

III BOBGA DOIR MASHQLAR

Tenglamalarni yeching (**204–205**):

204. a) $x^4-1=0$; | b) $5x^4-3x^3-4x^2-3x+5=0$; | c) $3x^4-4x^3-7x^2-4x+5=0$.

205. a) $(x-3)(x+14)(x-15)=0$; b) $(4x+11)(3x-5)=0$;
c) $x^4-15x^2-16=0$; d) $x^4+24x^2-25=0$.

Tengsizliklarni yeching (**206–208**):

206. a) $(2-x)(3x+1)(2x-3)>0$; | b) $(3x-2)(x-3)^3(x+1)^3(x+2)^4>0$.

207. a) $x^4+8x^3+12x^2\ge 0$; | b) $(16-x^2)(x^2+4)(x^2+x+1)(x^2-x-x)\le 0$.

208. a) $\dfrac{x^4-2x^2-8}{x^2+x-1}<0$; | b) $\dfrac{3x-2}{2x-3}<3$; | c) $\dfrac{7x-4}{x+2}\ge 1$; | d) $\dfrac{1}{x+1}+\dfrac{2}{x+3}<\dfrac{3}{x+2}$.

209. Tenglamalar sistemasini yeching:

a) $\begin{cases} x^2+y^2=113, \\ xy=56; \end{cases}$ b) $\begin{cases} x^2y+xy^2=84, \\ x^3+y^3=91; \end{cases}$

c) $\begin{cases} x^2+9xy+2y^2=12, \\ 2x^2+3xy-4y^2=1; \end{cases}$ d) $\begin{cases} x^2-2xy+3y^2=2, \\ x^2+xy+y^2=3. \end{cases}$

Tengsizliklar sistemasini yeching (**210–211**):

210. a) $\begin{cases} \dfrac{3x+5}{7}+\dfrac{10-3x}{5}>\dfrac{2x+7}{3}-7\dfrac{3}{21}, \\ \dfrac{7x}{3}-\dfrac{11(x+1)}{6}>\dfrac{3x-1}{3}-\dfrac{13-x}{2}; \end{cases}$ b) $\begin{cases} \dfrac{2x-11}{4}+\dfrac{19-2x}{2}<2x, \\ \dfrac{2x+15}{9}>\dfrac{x-1}{5}+\dfrac{x}{3}. \end{cases}$

211. a) $\begin{cases} 2x^2+2<5x, \\ x^2\ge x; \end{cases}$ b) $\begin{cases} x^2-16\le 0, \\ -x^2+16\ge 0; \end{cases}$ c) $\begin{cases} \dfrac{(x+2)(x^2-3x+8)}{x^2-9}\le 0, \\ \dfrac{1-x^2}{x^2+2x-8}\ge 0. \end{cases}$

212. Irratsional tenglamani yeching:

a) $\sqrt{8x+1}+\sqrt{3x-5}=\sqrt{7x+4}+\sqrt{2x-2}$;

b) $\sqrt{2x+3}+\sqrt{3x+2}-\sqrt{2x+5}=\sqrt{3x}$;

c) $\dfrac{\sqrt{3+2x}}{2x^2-x-1}>0$; d) $\sqrt{x-2}-\sqrt{x-3}>-\sqrt{x-5}$.

Sonlarni taqqoslang (**213–215**):

213. a) $4,2^{-\sqrt{2}}$ va 1; b) $0,2^{\frac{3}{5}}$ va $0,2^{-\frac{3}{5}}$; c) $(0,4)^{-\frac{\sqrt{5}}{2}}$ va 1.

214. a) $4^{0,5}$ va $4^{\frac{\sqrt{3}}{3}}$; b) $\sqrt{3}^{\,0,2}$ va $3^{0,2}$; c) $2^{-\frac{3}{4}}$ va $8^{-\frac{4}{9}}$.

215. a) $2^{-\sqrt{3}}$ va $2^{-\sqrt{5}}$; b) $7^{-0,3}$ va $7^{-\frac{1}{3}}$; c) $(\frac{1}{3})^{\sqrt{5}}$ va $3^{-\sqrt{3}}$.

216. Funksiyaning aniqlanish sohasini toping:

a) $y=5^{\sqrt{x^2-1}}$; b) $y=\dfrac{1}{3^x+1}$; c) $y=\dfrac{1}{3^{x^2}-9}$; d) $y=3^{\frac{1}{2-x}}$.

217. Funksiyaning qiymatlar sohasini toping:

a) $y=2^{-|x|}$; b) $y=3+4^{x+1}$; c) $y=-6^x$; d) $y=5^{|x|}+1$.

Tenglamalarni yeching (**218–219**):

218. a) $8^x=2^{\frac{1}{5}}$; | b) $121^x-7\cdot 11^x=5\cdot 11^x-11$; | c) $0,5^{\,x^2+x-3,5}=2\sqrt{2}$.

219. a) $6^{2x}-5^{2x-1}=6^{2x-1}+5^{2x}$; | b) $4^{x+3}+4^x=130$; | c) $125^x+20^x=2^{3x+1}$.

220. Tenglamalar sistemasini yeching:

a) $\begin{cases} x+y=5, \\ 5^{\,y-x^2}=0,2; \end{cases}$ b) $\begin{cases} 3^{x-1}=2^y, \\ 0,1^{2x-y}=0,01; \end{cases}$ c) $\begin{cases} 5^{x-y}=25, \\ 3^{x+y}=27. \end{cases}$

221. Tengsizlikni yeching:

a) $4^x\le 3^x$; | b) $16^x-7\cdot 4^x-8<0$; | c) $4^x\cdot 5^{1-x}<\dfrac{25}{4}$; | d) $6^{\frac{x-3}{x+8}}\ge 1$.

222. Sonlarni taqqoslang:

a) $\log_3 2$ va 2; b) $\log_3 5$ va $2\cdot\log_3 2$; c) $\log_2 5$ va $\log_5 2$

d) $\log_{0,2} 5$ va $\log_{0,2} 6$; e) $\log_4 3$ va $\log_3 4$; f) lg18,8 va lg6π.

223. Funksiyaning aniqlanish sohasini toping:

a) $y=\log_2(2x+7)$; | b) $y=\log_{\frac{1}{3}}(4-x^2)$; | c) $y=\log_5(-8x)$; | d) $y=\lg\dfrac{x-3}{x+8}$.

Tenglamalarni yeching:

224. a) $\lg(x-9)+\lg(2x-1)=2$; b) $\log_2\sqrt{x-3}+\log_2\sqrt{x+3}=2$.

Tenglamalar sistemasini yeching (**225–226**):

225. a) $\begin{cases} 5^{x-y}=1, \\ 2^{\log_2(x+y)}=6; \end{cases}$ b) $\begin{cases} \lg x+\lg y=4, \\ \lg x-\lg y=6; \end{cases}$ c) $\begin{cases} \log_{17}(3^x+2^y)=1, \\ 3^{x+1}-4\cdot 2^y=-5. \end{cases}$

226. a) $\begin{cases} 2^x\cdot 5^y=40, \\ 5^x\cdot 2^y=250; \end{cases}$ b) $\begin{cases} \log_2 x+5^{\log_5 y}=4, \\ x^y=16; \end{cases}$ c) $\begin{cases} 3^x\cdot 3^y=81, \\ 3^x-3^y=24. \end{cases}$

227. Tengsizlikni yeching:

a) $\log_3(x^2+x+1)\geq 1$; b) $\log_2(x^2+x-6)-\log_2(x+3)\leq 1$;

c) $\lg^2 x<\lg x^5-6$; d) $\log_3(4^x-5\cdot 2^x+13)>2$; e) $5^{x+7}>2$.

228. Funksiya grafigini chizing:

a) $y=1,5\sin(2x-1)$; b) $y=2\cos(2x-\frac{\pi}{3})$; c) $y=\log_3(1-x)$.

229. Taqqoslang:

a) $\arcsin(-\frac{1}{2})$ va $\arccos\frac{\sqrt{3}}{2}$; b) $\arccos\frac{1}{2}$ va $\mathrm{arctg}(-1)$;

c) $\mathrm{arc\,tg}\sqrt{3}$ va $\mathrm{arctg}1$; d) $\arccos\left(-\frac{1}{2}\right)$ va $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

230. Hisoblang:

a) $2\arcsin(-\frac{\sqrt{3}}{2})+\mathrm{arctg}(-1)+\arccos\frac{\sqrt{2}}{2}$;

b) $\mathrm{arctg}(-\sqrt{3})+\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arcsin 1$.

Tenglamani yeching (**231–233**):

231. a) $2\cos^2x+5\sin x-4=0$; b) $3\sin^2 2x+7\cos^2x-3=0$; c) $4\mathrm{tg}^2x-5\mathrm{tg}x+1=0$.

232. a) $3\sin^2x+7\sin x-10=0$; b) $2\cos^2x-5\cos x+3=0$; c) $\sin 6x=\sin 3x$.

233. a) $\cos 7x=\cos 2x$; b) $\mathrm{tg}8x=\mathrm{tg}11x$.

Tengsizlikni yeching (**234–235**):

234. a) $\sin x>-\frac{1}{2}$; b) $\cos 2x\leq\frac{1}{2}$; c) $\mathrm{tg}3x\geq 1$; d) $\sin 2x\leq\frac{1}{2}$.

235. a) $\sin 4x\leq\frac{1}{2}$; b) $\cos 10x\geq 0$; c) $\mathrm{tg}9x\leq\sqrt{3}$; d) $\cos\left(2x-\frac{\pi}{4}\right)\leq 0$.

Nazorat test topshiriqlari

1. Tenglamani yeching: $\sin 6x=0$.

A) $x=\frac{\pi}{6}n,\ n\in Z$; *B*) $x=\frac{\pi}{5}n,\ n\in Z$;

C) $x=\frac{\pi}{4}n,\ n\in Z$; *D*) $x=\frac{\pi}{3}n,\ n\in Z$.

2. Tenglamani yeching: $\cos 2x=0$.

A) $x=2\pi n,\ n\in Z$; B) $x=\pi n,\ n\in Z$;

C) $x=\frac{\pi}{4}+\frac{\pi}{2}n,\ n\in Z$; D) $x=\frac{\pi}{3}n,\ n\in Z$.

3. Tenglamani yeching: $\mathrm{tg}4x=\sqrt{3}$.

A) $x=\frac{\pi}{3}+\frac{\pi n}{4},\ n\in Z$; B) $x=\frac{\pi}{2}+\frac{\pi n}{4},\ n\in Z$;

C) $x=\frac{\pi}{12}+\frac{\pi n}{4},\ n\in Z$; D) $x=\frac{\pi n}{4},\ n\in Z$.

4. Tengsizlikni yeching: $\sin 2x>3$.

A) $x=\pi n,\ n\in Z$; | B) $\varnothing$; | C) $x=\frac{\pi}{2}+\pi n,\ n\in Z$; | D) $x=2\pi n,\ n\in Z$.

5. Tengsizlikni yeching: $\cos 2x<3$.

A) $(-\infty;+\infty)$; B) $\varnothing$; C) $(-\infty;0)$; D) $(0;+\infty)$.

6. Aniqlanish sohasini toping: $y=12^x$.

A) $(-\infty;+\infty)$; B) $(0;+\infty)$; C) $(-\infty;0)$; D) $\varnothing$.

7. Aniqlanish sohasini toping: $y=\log_2(3-x)$.

A) $(3;+\infty)$; B) $[3;+\infty)$; C) $(-\infty;3)$; D) $(-\infty;3]$.

8. Hisoblang: $\arcsin\frac{1}{2}$.

A) $\frac{\pi}{2}$; B) π; C) $\frac{\pi}{4}$; D) $\frac{\pi}{6}$.

9. Hisoblang: $\arccos\frac{\sqrt{3}}{2}$.

A) $\frac{\pi}{3}$; B) $\frac{\pi}{2}$; C) $\frac{\pi}{6}$; D) $\frac{\pi}{4}$.

10. Hisoblang: $\mathrm{arctg}1$.

A) $\frac{\pi}{3}$; B) $\frac{\pi}{2}$; C) $\frac{\pi}{6}$; D) $\frac{\pi}{4}$.

IV BOB

KOMPLEKS SONLAR

86-87 KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR. KOMPLEKS SONNI TASVIRLASH

Kompleks sonlar

Kompleks sonlar haqidagi ta'limot ilm-u fanda, xususan, matematikada alohida o'rin tutadi. Tez rivojlanayotgan bu soha texnikada, shuningdek, ishlab chiqarishning ko'plab sohalarida g'oyat keng qo'llanishga ega. Shu sonlar haqida ayrim ma'lumotlarni keltiramiz. Xususiy bir misoldan boshlaylik.

$x^2+4=0$ tenglamani yechish jarayonida $x_1=2\sqrt{-1}$ va $x_2=-2\sqrt{-1}$ "sonlar" hosil bo'ladi. Haqiqiy sonlar orasida esa bunday "sonlar" mavjud emas. Sunday holatdan qutulish uchun $\sqrt{-1}$ ga *son* deb qarash zarurati paydo bo'ladi.

Bu yangi son hech qanday real kattalikning o'lchamini yoki uning o'zgarishini ifodalamaydi. Shu sababli $\sqrt{-1}$ ni ***mavhum*** (hayoliy, haqiqatda mavjud bo'lmagan) ***birlik*** deb atash va maxsus belgilash qabul qilingan: $\sqrt{-1}=i$. Mavhum birlik uchun $i^2=-1$ tenglik o'rinlidir.

$a+bi$ ko'rinishdagi ifodani qaraymiz. Bu yerda a va b lar istalgan haqiqiy sonlar, i esa mavhum birlik.

$a+bi$ ifoda haqiqiy son a va mavhum son bi lar "kompleksi"dan iborat bo'lgani uchun uni kompleks son deb atash qabul qilingan.

$a+bi$ ifoda algebraik shakldagi kompleks son deb ataladi.

$a+bi$ ni "algebraik shakldagi kompleks son" deyish o'rniga qisqalik uchun "kompleks son" deb ataymiz. Kompleks sonlarni bitta harf bilan belgilash qulay. Masalan, $a+bi$ ni z bilan belgilaylik. $z=a+bi$ kompleks sonning haqiqiy qismi a ni Re(z) (fransuzcha réelle – haqiqiy) kabi, mavhum qismi b ni esa Im(z) (fransuzcha *imaginaire* – mavhum) kabi belgilash qabul qilingan: a=Re(z), b=Im(z).

Agar $z=a+bi$ kompleks son uchun $b=0$ bo'lsa, haqiqiy son $z=a$ hosil bo'ladi.

Demak, haqiqiy sonlar to‘plami R barcha kompleks sonlar to‘plami C ning qism to‘plami bo‘ladi: $R \subset C$.

1- misol. $z_1=1+2i$, $z_2=2-i$, $z_3=2{,}1$, $z_4=2i$, $z_5=0$ kompleks sonlarning haqiqiy va mavhum qismlarini toping.

△ Kompleks sonlarning haqiqiy va mavhum qismlarining ta’riflariga ko‘ra, topamiz:

$\mathrm{Re}(z_1)=1$; $\mathrm{Re}(z_2)=2$; $\mathrm{Re}(z_3)=2{,}1$; $\mathrm{Re}(z_4)=0$; $\mathrm{Re}(z_5)=0$;

$\mathrm{Im}(z_1)=2$; $\mathrm{Im}(z_2)=-1$; $\mathrm{Im}(z_3)=0$; $\mathrm{Im}(z_4)=2$; $\mathrm{Im}(z_5)=0$. ▲

Kompleks sonlar uchun “<”, “>” munosabatlari aniqlanmagan, lekin teng kompleks sonlar tushunchasi kiritiladi.

Ikkita komlpeks sonning haqiqiy va mavhum qismlari, mos ravishda, teng bo‘lsa, bunday kompleks sonlar o‘zaro teng deb ataladi.

Masalan, $z_1=1{,}5+\frac{4}{5}i$ va $z_2=\frac{3}{2}+0{,}8i$ sonlari uchun $\mathrm{Re}(z_1)=\mathrm{Re}(z_2)=1{,}5$; $\mathrm{Im}(z_1)=\mathrm{Im}(z_2)=0{,}8$. Demak, $z_1=z_2$.

Bir-biridan faqat mavhum qismlarining ishorasi bilan farq qiladigan ikki kompleks son o‘zaro qo‘shma kompleks sonlar deyiladi. $z=a+bi$ kompleks songa qo‘shma kompleks son $\bar{z}=a-bi$ ko‘rinishda yoziladi.

Masalan, $6+7i$ va $6-7i$ lar qo‘shma kompleks sonlardir: $\overline{6+7i}=6-7i$. Shu kabi $\bar{z}$ soniga qo‘shma son $\bar{\bar{z}}=z$ bo‘ladi. Masalan, $\overline{\overline{6+7i}}=\overline{6-7i}=6+7i$. a haqiqiy songa qo‘shma son a ning o‘ziga teng: $\bar{a}=\overline{a+0\cdot i}=a-0\cdot i=a$. Lekin, bi mavhum songa qo‘shma son $\overline{bi}=-bi$ dir. Chunki $\overline{bi}=\overline{0+bi}=0-bi=-bi$.

Kompleks sonlar ustida arifmetik amallar

Kompleks sonlar ustida arifmetik amallar quyidagicha aniqlanadi:

$$(a+bi)+(c+di)=(a+c)+(b+d)i; \quad (1)$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i; \quad (2)$$

$$(a+bi)\cdot(c+di)=(ac-bd)+(ad+bc)i; \quad (3)$$

$$\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i. \quad (4)$$

(1) va (2) tengliklarni bevosita qo‘llash qiyin emas. Kompleks sonlarni ko‘paytirish amalini $i^2=-1$ ekanligini e’tiborga olib, ko‘phadlarni ko‘paytirish kabi bajarish mumkin.

2- misol. Yig‘indini toping: $(3+7i)+(-5+4i)$.

△ Yig‘indini topish uchun (1) formuladan foydalanamiz:

$(3+7i)+(-5+4i)=(3+(-5))+(7+4)i=-2+11i$. ▲

3- misol. Ayirmani toping: $(13-7i)-(-5+4i)$.

△Ayirmani topish uchun (2) formuladan foydalanamiz:
$(13-7i)-(-5+4i)=(13-(-5))+(-7-4)i=18-11i$. ▲

4- misol. Ko‘paytmani toping: $(2-i)\cdot(\frac{3}{4}+2i)$.

△Qavslarni ochamiz va $i^2=-1$ ekanidan foydalanamiz:

$$(2-i)\cdot\left(\frac{3}{4}+2i\right)=2\cdot\frac{3}{4}+2\cdot 2i-i\cdot\frac{3}{4}-2i^2=\frac{3}{2}+4i-\frac{3}{4}i+2=\frac{7}{2}+\frac{13}{4}i. ▲$$

$\frac{a+bi}{c+di}$ bo‘linmani hisoblash uchun uning surati va maxrajini maxrajning "qo‘shmasi" $c-di$ ga ko‘paytirib, tegishli amallarni bajarish lozim.

5- misol. Bo‘lish amalini bajaring: $\frac{2-i}{-3+2i}$.

$$△\ \frac{2-i}{-3+2i}=\frac{(2-i)(-3-2i)}{(-3+2i)(-3-2i)}=\frac{-6-4i+3i-2}{(-3)^2-(2i)^2}=\frac{-8-i}{13}=\frac{-8}{13}-\frac{1}{13}i. ▲$$

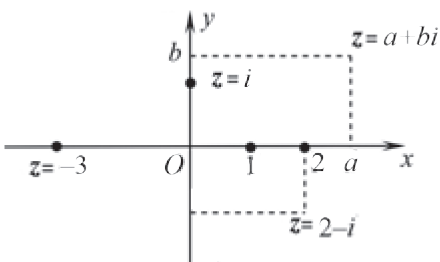
$z+w=0$ tenglikni qanoatlantiruvchi z, w kompleks sonlar o‘zaro *qarama-qarshi* sonlar deyiladi. z kompleks soniga qarama-qarshi sonni $-z$ bilan belgilash qabul qilingan. $z=a+bi$ kompleks songa qarama-qarshi bo‘lgan yagona kompleks son mavjud va bu son $-z=-a-bi$ kompleks sondan iborat.

$zw=1$ tenglikni qanoatlantiradigan z va w kompleks sonlar *o‘zaro teskari* kompleks sonlar deyiladi. $z=0$ songa teskari son mavjud emas. Har qanday $z\neq0$ kompleks songa teskari kompleks son mavjud. Bu son $\frac{1}{z}$ sonidan iborat.

Kompleks sonni tekislikda tasvirlash

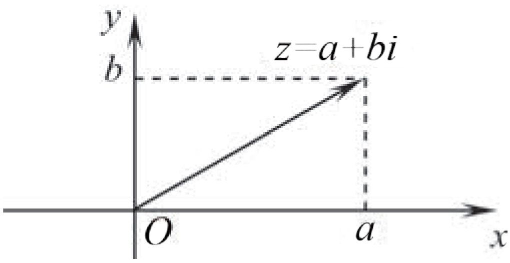
Faraz qilaylik, tekislikda to‘g‘ri burchakli Dekart koordinatalar sistemasi berilgan bo‘lsin. U holda $z=a+bi$ kompleks songa tekislikda koordinatalari $(a; b)$ bo‘lgan nuqta mos keladi.

Bu usul bilan tasvirlashda $a+0i$ kompleks songa $(a; 0)$ koordinatali nuqta, $0+bi$ kompleks songa esa $(0; b)$ nuqta mos keladi. Shuning uchun ham Ox o‘q haqiqiy va Oy o‘q mavhum o‘q deyiladi (1- rasm).

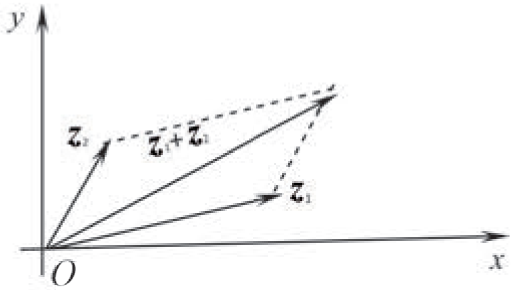


1- rasm.

$a+bi$ kompleks sonni tekislikda a va b koordinatali vektor kabi ham tasvirlash mumkin (2- rasm). Bu esa kompleks sonlarni qo‘shishda vektorlarni qo‘shishning parallelogramm qoidasini qo‘llash imkonini beradi (3- rasm).



2- rasm.



3- rasm.

Savol va topshiriqlar

1. Mavhum birlik nima? Nega uni kiritishga ehtiyoj sezildi?
2. Kompleks sonning algebraik ko‘rinishini yozing, misol keltiring
3. Ikkita kompleks son qachon teng deyiladi? Misol keltiring.
4. Ikkita kompleks sonning yig‘indisi, ayirmasi, ko‘paytmasi, bo‘linmasi qanday aniqlanadi? Misollarda tushuntiring.
5. Qarama-qarshi kompleks son nima?
6. Qo‘shma kompleks son nima?
7. O‘zaro teskari ko‘mpleks sonlar nima? Misollar keltiring.
8. Kompleks sonni vektor kabi tasvirlash nima? Misol keltiring.

Mashqlar

1. Kompleks sonlarning haqiqiy va mavhum qismlarini ayting:

1) $z=-3+7i$; 2) $z=4-\frac{1}{2}i$; 3) $z=-2-5i$;

4) $z=-5,7+5i$; 5) $z=5i$; 6) $z=9$;

7) $z=-7+3i$; 8) $z=8-\frac{1}{2}i$; 9) $z=-5-6i$;

10) $z=-5,7-5i$; 11) $z=-5i$; 12) $z=90$.

2. Kompleks sonlarni algebraik ko‘rinishda yozing:

1) $\mathrm{Re}(z)=4,\quad \mathrm{Im}(z)=-5$; 2) $\mathrm{Re}(z)=-2,\quad \mathrm{Im}(z)=3$;

3) $\mathrm{Re}(z)=0,\quad \mathrm{Im}(z)=8$; 4) $\mathrm{Re}(z)=7,\quad \mathrm{Im}(z)=0$;

5) $\mathrm{Re}(z)=6,\quad \mathrm{Im}(z)=-7$; 6) $\mathrm{Re}(z)=-3,\quad \mathrm{Im}(z)=4$;

7) $\mathrm{Re}(z)=0,\quad \mathrm{Im}(z)=9$; 8) $\mathrm{Re}(z)=2,\quad \mathrm{Im}(z)=0$;

9) $\mathrm{Re}(z)=12,\quad \mathrm{Im}(z)=20$.

Teng kompleks sonlarni ko'rsating (**3–4**):

3. 1) $2-4i$; | 2) $2+3i$; | 3) $\frac{2}{3}+i$; | 4) $\sqrt{121}-7i$; | 5) $33+44i$; | 6) $\sqrt[3]{8}+\sqrt[3]{27}\,i$.

4. 1) $4-3i$; | 2) $1+3i$; | 3) $\frac{1}{3}+i$; | 4) $\sqrt{16}-\sqrt{9}i$; | 5) $3+4i$; | 6) $\sqrt[3]{27}+\sqrt[3]{64}i$.

z soniga qo'shma bo'lgan $\bar{z}$ sonni toping (**5–6**):

5. 1) $z=5-3i$; | 2) $z=-5+3i$; | 3) $z=1-i$; | 4) $z=2+3i$; | 5) $z=-7-i$.

6. 1) $z=7{,}2$; | 2) $z=6i$; | 3) $z=\sqrt{16}-\sqrt{9}i$; | 4) $z=-2i+(-7+3i)$.

7. Yig'indini toping (**7–8**):
1) $(-5+3i)+(2-i)$; | 2) $(-3)+(3-4i)$; | 3) $(2+5i)+(-2-5i)$; | 4) $(-4i)+(3.6-3i)$.

8. 1) $(8-3i)+(8+3i)$; | 2) $(-7+5i)+(7-5i)$; | 3) $9i+(3-8i)$; | 4) $-17i+(-9+16i)$.

Ayirmani toping (**9–10**):

9. 1) $(3+4i)-(4+2i)$; | 2) $(4-6i)-(3+2i)$; | 3) $(2+4i)-(-4+2i)$.

10. 1) $(5+4i)-(5-4i)$; | 2) $7-(8+5i)$; | 3) $7i-(6i+3)$.

Ko'paytmani toping (**11–12**):

11. 1) $(4+6i)(3+4i)$; | 2) $(5+8i)(3-2i)$; | 3) $(6-4i)(3-6i)$.

12. 1) $(-3+2i)(8-4i)$; | 2) $\left(\frac{1}{3}-i\right)\left(\frac{1}{2}+i\right)$; | 3) $\left(\frac{5}{7}+4i\right)\left(\frac{7}{5}-2i\right)$.

Bo'linmani toping (**13–14**):

13. 1) $\frac{2+2i}{1-2i}$; | 2) $\frac{4-5i}{3+2i}$; | 3) $\frac{3+4i}{3-4i}$; | 4) $\frac{2+3i}{4-3i}$; | 5) $\frac{4-5i}{3+2i}$.

14. 1) $\frac{4-5i}{-2+3i}$; | 2) $\frac{3}{5-2i}$; | 3) $\frac{5-2i}{3}$; | 4) $\frac{7i}{13-i}$; | 5) $\frac{7+4i}{5-6i}$.

Amallarni bajaring (**15–16**):

15. 1) $\frac{(3-4i)(4-3i)}{2+i}$; | 2) $\frac{(4-i)(3+2i)}{3-2i}$; | 3) $\frac{5-2i}{(2+i)(1-i)}$.

16. 1) $\frac{3-2i}{(1+i)(3-i)}$; | 2) $\frac{3}{2-3i}+\frac{3}{2+3i}$; | 3) $\frac{2}{1+i}+\frac{5}{2+i}$.

Kompleks sonlarni tekislikda tasvirlang (**17–18**):

17. 1) $z=3+4i$; | 2) $z=3-4i$; | 3) $z=-3+4i$; | 4) $z=-3-4i$; | 5) $z=2i$.

18. 1) $z=4-2i$; | 2) $z=5+3i$; | 3) $z=\frac{2+i}{2-i}$; | 4) $z=(2-i)(1+i)$; | 5) $z=(2+i)(2-i)$.

88 $r(\cos\varphi + i\sin\varphi)$ va $r\cdot e^{i\varphi}$ $(r>0,\ 0\le\varphi\le 2\pi)$ KO'RINISHDAGI KOMPLEKS SONLAR

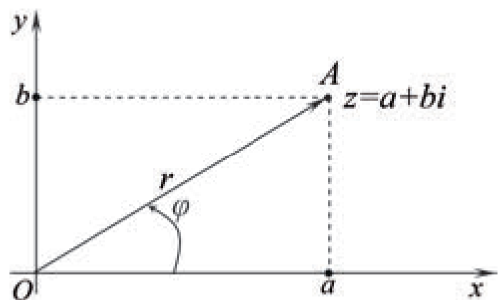
Bu mavzuda kompleks sonning trigonometrik va ko'rsatkichli ko'rinishlarini o'rganamiz.

Trigonometrik ko'rinishdagi kompleks sonlar

Tekislikda to'g'ri burchakli Dekart koordinatalar sistemasi berilgan bo'lsin. $z=a+bi$ kompleks songa $(a;\ b)$ koordinatali A nuqta mos qo'yilgan, deylik. Koordinatalar boshi O va A nuqtalarini tutashtirib $\overrightarrow{OA}$ vektorni hosil qilamiz (4-rasm).

O nuqtadan A nuqtagacha bo'lgan $r=OA$ masofa ***kompleks sonning moduli***, abssissa o'qining musbat yo'nalishi hamda $\overrightarrow{OA}$ vektor orasidagi (φ) burchak ***kompleks sonning argumenti*** deyiladi.

Ravshanki, $0\le r<+\infty$, $0\le\varphi<2\pi$, $r=\sqrt{a^2+b^2}$, $\cos\varphi=\dfrac{a}{r}$, $\sin\varphi=\dfrac{b}{r}$.



4- rasm.

Kompleks sonning $z=r(\cos\varphi+i\sin\varphi)$ ko'rinishiga uning trigonometrik shakli va $z=r\cdot e^{i\varphi}$ ko'rinishiga esa ko'rsatkichli shakli deyiladi. Kompleks sonni trigonometrik ko'rinishidan algebraik ko'rinishiga o'tkazish uchun quyidagi formuladan foydalanadi: $a=r\cos\varphi$, $b=r\sin\varphi$.

1-misol. Kompleks sonlarni trigonometrik ko'rinishda yozing:

1) i; 2) $-2i$; 3) $-1-i$.

△ 1) $z=i=0+1\cdot i$, $a=0$, $b=1$, $r=\sqrt{0^2+1^2}=1$, $\cos\varphi=\dfrac{0}{1}=0$, $\varphi=\dfrac{\pi}{2}$.

Demak, $i=1\left(\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}\right)$, ya'ni $i=\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$.

2) $r=2$, $\varphi=\dfrac{3\pi}{2}$ bo'lganligi uchun $-2i=2\left(\cos\dfrac{3\pi}{2}+i\sin\dfrac{3\pi}{2}\right)$;

3) $z=-1-i$, $a=-1$, $b=-1$, $r=\sqrt{2}$, $\cos\varphi=-\dfrac{1}{\sqrt{2}}$, $\varphi=\dfrac{5\pi}{4}$.

Demak, $-1-i=\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$. ▲

2-misol. Kompleks sonlarni ko'rsatkichli ko'rinishda yozing:

1) i; 2) $-2i$; 3) $-1-i$.

△ 1-misolning hisoblashlaridan foydalanamiz:

$$i=e^{\frac{\pi}{2}i},\qquad -2i=2e^{\frac{3\pi}{2}i},\qquad -1-i=\sqrt{2}e^{\frac{5\pi}{4}i}.$$ ▲

Savol va topshiriqlar

1. Kompleks sonning moduli nima? U qanday hisoblanadi?
2. Kompleks sonning argumenti nima? Misol keltiring.
3. Kompleks sonning trigonometrik ko'rinishini tushuntiring.
4. Kompleks sonning ko'rsatkichli ko'rinishini tushuntiring.
5. *Eylerning mashhur formulasini* isbotlang: $e^{i\pi}=-1$.

Mashqlar

Kompleks sonning modulini toping (**19–20**):

19. 1) $z=-2+3i$; 2) $z=-2+3i$; 3) $z=1+\sqrt{3}i$; 4) $z=\sqrt{8}-i$.

20. 1) $z=6-8i$; 2) $z=2+2\sqrt{3}i$; 3) $z=\sqrt{3}+i$; 4) $z=2i$.

Kompleks sonning argumentini toping (**21–22**):

21. 1) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$; 2) $z=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$; 3) $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$; 4) $z=2\sqrt{2}i$.

22. 1) $z=5$; 2) $z=-2i$; 3) $z=\frac{\sqrt{33}}{2}-\frac{\sqrt{11}}{2}i$.

Kompleks sonni trigonometrik va ko'rsatkichli ko'rinishda yozing (**23–24**):

23. 1) $z=-2-2i$; | 2) $z=2-2i$; | 3) $z=\sqrt{3}-i$; | 4) $z=1-\sqrt{3}i$.

24. 1) $z=\sqrt{2}-\sqrt{2}i$; | 2) $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$; | 3) $z=\frac{\sqrt{33}}{2}-\frac{\sqrt{11}}{2}i$; | 4) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$.

89-90 TRIGONOMETRIK SHAKLDA BERILGAN KOMPLEKS SONLARNING KO'PAYTMASI VA BO'LINMASI

Trigonometrik shaklda berilgan kompleks sonlarni ko'paytirish

$z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$, $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ trigonometrik korinishdagi kompleks sonlarning ko'paytmasi uchun quyidagi formula o'rinli:

$$z_1\cdot z_2=r_1\cdot r_2\left[\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2)\right]. \tag{1}$$

z_1 va z_2 trigonometrik ko‘rinishdagi sonlarni bo‘lish uchun esa

$$\frac{z_2}{z_1}=\frac{r_2}{r_1}\left[\cos\left(\varphi_2-\varphi_1\right)+i\sin\left(\varphi_2-\varphi_1\right)\right] \text{formula o‘rinli, } r_1\neq 0. \qquad (2)$$

1- misol. $z_1=3\left(\cos 20°+i\sin 20°\right)$ va $z_2=2\left(\cos 35°+i\sin 35°\right)$ kompleks sonlarni ko‘paytiring.

△ Yuqoridagi qoidaga ko‘ra, ko‘paytmani topamiz:

$z_1\cdot z_2=3\cdot 2(\cos(20°+35°)+i\sin(20°+35°))=6\left(\cos 55°+i\sin 55°\right)$. ▲

2- misol. $z_1=2\left(\cos 140°+i\sin 140°\right)$, $z_2=3\left(\cos 150°+i\sin 150°\right)$ va

va $z_3=5\left(\cos 70°+i\sin 70°\right)$ kompleks sonlarni ko‘paytiring.

△ Yuqoridagi qoidaga ko‘ra ko‘paytmani topamiz:

$$z_1\cdot z_2\cdot z_3=2\cdot 3\cdot 5\left[\cos\left(140°+150°+70°\right)+i\sin\left(140°+150°+70°\right)\right]=$$

$$=30\left(\cos 360°+i\sin 360°\right)=30. ▲$$

3- misol. $z_1=6\left(\cos 50°+i\sin 50°\right)$ va $z_2=2\left(\cos 25°+i\sin 25°\right)$ kompleks sonlar bo‘linmasini toping.

△ Bo‘lishning qoidasiga muvofiq:

$$\frac{z_1}{z_2}=\frac{6}{2}\left[\cos\left(50°-25°\right)+i\sin\left(50°-25°\right)\right]=3\left(\cos 25°+i\sin 25°\right). ▲$$

Natural darajaga ko‘tarish

$z=r\left(\cos\varphi+i\sin\varphi\right)$ kompleks sonni kvadratga ko‘tarish uchun kompleks sonlarni ko‘paytirish formulasi (1) dan foydalanamiz:

$z^2=r^2\left(\cos\varphi+i\sin\varphi\right)^2=r^2\left(\cos 2\varphi+i\sin 2\varphi\right)$. Shuningdek,

$$z^3=\left[r\left(\cos\varphi+i\sin\varphi\right)\right]^3=r^3(\cos 3\varphi+i\sin 3\varphi). \qquad (3)$$

Umuman, *Muavr formulasi* deb ataladigan ushbu

$z^n=\left[r\left(\cos\varphi+i\sin\varphi\right)\right]^n=r^n(\cos n\varphi+i\sin n\varphi)$ formula o‘rinli, bunda $n\in N$.

4- misol. $z=3\left(\cos 15°+i\sin 15°\right)$ kompleks sonni kubga ko‘taring:

△ (3) formulaga ko‘ra:

$$z^3=27\left(\cos 45°+i\sin 45°\right)=\frac{27}{2}\left(\sqrt{2}+i\sqrt{2}\right)=\frac{27}{\sqrt{2}}(1+i). ▲$$

5-misol. $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ kompleks sonning 10- darajasini toping.

△ Avval berilgan sonning moduli va argumentini topib, uni trigonometrik ko‘rinishda yozib olamiz: $r=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$, $\varphi=\frac{\pi}{3}=60°$, $z=1\cdot\left(\cos 60°+i\sin 60°\right)$, bu yerdan:

$$z^{10}=\left(\cos 600°+i\sin 600°\right)=\cos 240°+i\sin 240°=-\frac{1}{2}-\frac{\sqrt{3}}{2}i\,.$$ ▲

Savol va topshiriqlar

1. Trigonometrik ko‘rinishdagi kompleks sonlar qanday ko‘paytiriladi? Ma’nosini oching va ayting.
2. Trigonometrik ko‘rinishdagi kompleks sonlar qanday bo‘linadi? Ma’nosini oching va ayting.
3. Trigonometrik ko‘rinishdagi kompleks sonlar darajaga qanday ko‘tariladi?

Mashqlar

Kompleks sonlarni ko‘paytiring (**27–28**):

27. 1) $z_1=\frac{\sqrt{3}}{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$ va $z_2=\frac{1}{2}(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$;

2) $z_1=\frac{1}{3}(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9})$ va $z_2=3(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$.

28. 1) $z_1=\frac{1}{\sqrt{3}}(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})$ va $z_2=\sqrt{3}(\cos\pi+i\sin\pi)$;

2) $z_1=2(\cos\frac{\pi}{18}+i\sin\frac{\pi}{18})$ va $z_2=\frac{1}{2}(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$.

Kompleks sonlarni bo‘ling (**29–30**):

29. 1) $z_1=\sqrt{2}(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8})$ ni $z_2=2(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})$ ga;

2) $z_1=8(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$ ni $z_2=4(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$ ga.

30. 1) $z_1=\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}$ ni $z_2=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ ga;

2) $z_1=\frac{\sqrt{3}}{2}(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3})$ ni $z_2=\frac{2}{\sqrt{3}}(\cos\frac{2\pi}{3}+)\sin\frac{2\pi}{3})$ ga.

Kompleks sonni darajaga ko‘taring (**31–32**):

31. 1) $(3(\cos\frac{\pi}{15}+i\sin\frac{\pi}{15}))^5$; 2) $(\sqrt{3}(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}))^6$; 3) $(\sqrt{2}(\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}))^7$.

32. 1) $(4(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}))^4$; 2) $(\cos\frac{\pi}{15}+i\sin\frac{\pi}{15})^{10}$; 3) $(\cos\frac{\pi}{22}+i\sin\frac{\pi}{22})^{11}$.

Amallarni bajaring (**33–34**):

33. 1) $\dfrac{(1+i)^5(\sqrt{2}-i)^4}{(1-i)(1+\sqrt{2}\,i)^4}$; 2) $\dfrac{(1-i)^4(\sqrt{2}+i)^3}{(1+i)^4}$; 3) $\dfrac{(1+i)^{15}}{(1-i)^{10}-(1+i)^{10}\cdot i}$.

34. 1) $\dfrac{2+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; 2) $\dfrac{12+5i}{6-8i}+\dfrac{(2-i)^2}{1-2i}$; 3) $\dfrac{3-4i}{3+4i}+\dfrac{5+6i}{5-6i}$.

91 KOMPLEKS SONDAN KVADRAT ILDIZ CHIQARISH

$z=r(\cos\varphi+i\sin\varphi)$ korinishdagi kompleks sondan kvadrat ildiz chiqarish uchun izlanayotgan kompleks sonning modulini x va argumentini y deb quyidagi tenglikni yozamiz:

$$\sqrt{r(\cos\varphi+i\sin\varphi)}=x(\cos y+i\sin y).$$

Tenglikning ikkala qismini kvadratga ko‘tarib, $r(\cos\varphi+i\sin\varphi)=x^2(\cos 2y+i\sin 2y)$ hamda $x^2=r$, $2y=\varphi+2\pi n$ ekanidan $x=\sqrt{r}$, $y=\dfrac{\varphi}{2}+\pi n$, $n\in\mathbb{Z}$ munosabatlarni topamiz. Demak, izlanayotgan z kompleks sonning kvadrat ildizi uchun

$$\beta=\sqrt{r}\left[\cos\frac{\varphi+2\pi n}{2}+i\sin\frac{\varphi+2\pi n}{2}\right]$$

formula o‘rinli. n ga $0,\ \pm1,\ \pm2,\ \ldots$ qiymatlarni qo‘yib, turli ildizlarni topamiz. Tekshirish natijasida faqat 2 ta turli qiymat borligi aniqlanadi, ya’ni

$$n=0 \text{ da } \beta_1=\sqrt{r}\left(\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}\right), \qquad (1)$$

$$n=1 \text{ da } \beta_2=\sqrt{r}\left[\cos\left(\frac{\varphi}{2}+\pi\right)+i\sin\left(\frac{\varphi}{2}+\pi\right)\right]. \qquad (2)$$

1- misol. $z=9(\cos 60°+i\sin 60°)$ kompleks sondan kvadrat ildiz chiqaring.

△ Yuqoridagi formulaga ko‘ra, kvadrat ildizlarni hisoblaymiz:

$$\sqrt{z}=3[\cos(30°+180°n)+i\sin(30°+180°n)].$$

Bu formulada

$n=0$ uchun $\sqrt{z}=3(\cos 30°+i\sin 30°)=\dfrac{3}{2}(\sqrt{3}+i)$ va

$n=1$ uchun $\sqrt{z}=3(\cos 210°+i\sin 210°)=-\dfrac{3}{2}(\sqrt{3}+i)$ kvadrat ildizlar topiladi. ▲

Kompleks sondan kub ildiz chiqarishda quyidagi formuladan foydalaniladi:

$$z_n=\sqrt[3]{r(\cos\varphi+i\sin\varphi)}=x(\cos y+i\sin y)=\sqrt[3]{r}(\cos\frac{\varphi+360°n}{3}+i\sin\frac{\varphi+360°n}{3}),$$

$n=0, 1, 2.$

Bu topilgan sonlar Dekart koordinatalar sistemasida markazi koordinata boshida va radiusi $\sqrt[3]{r}$ bo'lgan aylanaga ichki chizilgan muntazam uchburchak uchlaridan iboratdir.

2- misol. $z=1$ ko'mpleks sonning kub ildizini toping va chizmada ko'rsating.

△Bu sonning moduli $r=1$ va argumenti $\varphi=0°$ bo'lgani uchun

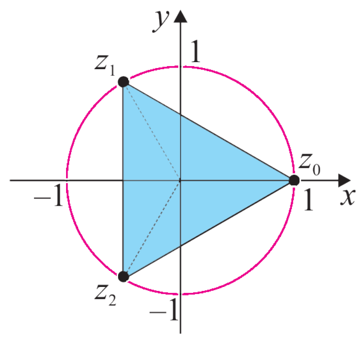
$$z_n=\sqrt[3]{1}=1\cdot\left(\cos\frac{0°+360°n}{3}+i\sin\frac{0°+360°n}{3}\right),\ n=0,\ 1,\ 2.$$

Bu yerdan: $n=0$ da $z_0=1\cdot(\cos 0°+i\sin 0°)=1,$

$n=1$ da $z_1=1\cdot(\cos 120°+i\sin 120°)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i,$

$n=2$ da $z_2=1\cdot(\cos 240°+i\sin 240°)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i.$

Bu sonlar muntazam uchburchakning uchlaridan iborat ekanini 5- rasmdan ko'rishimiz mumkin.



5- rasm.

Kompleks sondan 4- darajali ildiz chiqarishda quyidagi formuladan foydalaniladi:

$$z_n=\sqrt[4]{r(\cos\varphi+i\sin\varphi)}=\sqrt[4]{r}\left(\cos\frac{\varphi+360°n}{4}+i\sin\frac{\varphi+360°n}{4}\right),$$

bu yerda $n=0, 1, 2, 3.$

3- misol. $z=i$ kompleks sondan 4- darajali ildiz chiqaring.

△Bu sonning moduli $r=1$, argumenti $\varphi=90°$ bo'lgani uchun

$$z_n=\sqrt[4]{1(\cos 90°+i\sin 90°)}=1\cdot\left(\cos\frac{90°+360°n}{4}+\sin\frac{90°+360°n}{4}\right).$$

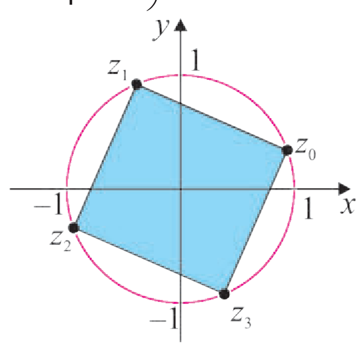
Bu yerdan: $n=0$ da $z_0=\cos 22{,}5°+i\sin 22{,}5°,$

$n=1$ da $z_1=\cos 112{,}5°+i\sin 112{,}5°,$

$n=2$ da $z_2=\cos 202{,}5°+i\sin 202{,}5°,$

$n=3$ da $z_3=\cos 292{,}5°+i\sin 292{,}5°.$

Bu sonlar markazi koordinata boshida va radiusi 1 bo'lgan aylanaga ichki chizilgan kvadratning uchlaridan iboratdir (6- rasm).



6- rasm

Savol va topshiriqlar

1. Kompleks sondan kvadrat ildiz qaysi formula orqali topiladi?
2. Muavr formulasi nima? Uning ma'nosini oching va ayting.

Mashqlar

Kompleks sondan kvadrat ildiz chiqaring (**35–36**):

35. 1) $z=25\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$; 2) $z=\frac{1}{4}\left(\cos\frac{\pi}{18}+i\cdot\sin\frac{\pi}{18}\right)$;

3) $z=\cos\frac{\pi}{5}+i\cdot\sin\frac{\pi}{5}$; 4) $z=\cos\frac{3\pi}{4}+i\cdot\sin\frac{3\pi}{4}$;

5) $z=2\left(\cos\frac{\pi}{30}+i\cdot\sin\frac{\pi}{30}\right)$; 6) $z=\frac{1}{49}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

7) $z=\cos\frac{\pi}{10}+i\cdot\sin\frac{\pi}{10}$; 8) $z=\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}$.

36. 1) $z=2\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$; 2) $z=\frac{1}{2}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

3) $z=\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$; 4) $z=\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}$;

5) $z=2\left(\cos\frac{\pi}{2}+i\cdot\sin\frac{\pi}{2}\right)$; 6) $z=\frac{16}{9}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

7) $z=5(\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4})$; 8) $z=\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$.

IV BOBGA DOIR MASHQLAR

37. Hisoblang:

1) $(3+4i)(2-5i)+(3-4i)(2+5i)$; 2) $(1+3i)^3-\left(4+i^5\right)$;

3) $\frac{(1-2i)^2}{1+3i}$; 4) $5-7i+8i^2-9i^3+i^4$.

38. Algebraik ko'rinishda yozing:

1) $z=\left(\frac{1-\sqrt{3}\,i}{3i}\right)^2$; 2) $z=\frac{12-13i}{8+6i}+\frac{(1+2i)^2}{i+3}$; 3) $\frac{4i}{(\sqrt{3}-i)^2}$.

Hisoblang (**39–42**):

39. 1) $(1+i)^{10}$; 2) $(1-i)^4\left(-2\sqrt{3}+2i\right)^3$; 3) $(1+i)^{2018}\cdot(1-1)^{2018}$;

4) $\left(\dfrac{\sqrt{3}+i}{1-i}\right)^8$; 5) $\dfrac{2\sqrt{3}-2i}{(-1+i)\left(\sqrt{2}+\sqrt{6}\,i\right)}$; 6) $\left(\dfrac{\sqrt{2}-i}{1+i}\right)^{10}$.

40. 1) $z=\dfrac{(2+i)^2}{3-4i}$; 2) $z=\dfrac{(1+2i)^3}{2i}-3i^{10}$; 3) $z=\left(\dfrac{1-i}{1+i}\right)^5$;

4) $z=\dfrac{3+2i}{1+4i}-i^7$; 5) $\dfrac{(4-i)}{3+4i}$; 6) $\dfrac{2-3i}{1-4i}$.

41. 1) $\dfrac{2+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; 2) $\dfrac{12+5i}{6-8i}+\dfrac{(2-i)^2}{1-2i}$;

3) $(2-3i)^3-(2+3i)^3$; 4) $\dfrac{(4+3i)(2+3i)^2}{6+8i}$;

5) $\dfrac{33+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; 6) $\dfrac{12-5i}{6-8i}+\dfrac{(2+i)^2}{1-2i}$.

42. 1) $(2-2i)\cdot 2\sqrt{3}\left(\cos 70^\circ+i\sin 70^\circ\right)$; 2) $\left(\cos\left(-\dfrac{\pi}{6}\right)+i\sin\left(-\dfrac{\pi}{6}\right)\right)\cdot\left(\sqrt{3}-3i\right)$.

43. Bo‘lishni bajaring:

1) $5\left(\cos 100^\circ+i\sin 100^\circ\right):\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)$; 2) $(6+6i):3\left(\cos 75^\circ+i\sin 75^\circ\right)$.

44. Darajaga ko‘taring:

1) $\left(1-\sqrt{3}i\right)^3$; 2) $\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)^4$; 3) $\left(\dfrac{1}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}i\right)^6$;

4) $\left(1-\sqrt{3}i\right)^5$; 5) $\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)^{10}$; 6) $\left(\dfrac{1}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}i\right)^{10}$.

45. Kvadrat ildizni hisoblang:

1) $\sqrt{-27i}$; 2) $\sqrt{6-6\sqrt{3}\,i}$; 3) $\sqrt{8+8\sqrt{3}\,i}$; 4) $\sqrt{-256}$.

46. Tenglikni tekshiring:

1) $$\left[\frac{-\sqrt{3}+i}{2}\right]^5+\left[\frac{-\sqrt{3}-i}{2}\right]^5=\sqrt{3};$$

2) $$\frac{\left(\sin 26^\circ+i\cos 154^\circ\right)\cdot\left(\sin 27^\circ+i\cos 153^\circ\right)^3}{\sin 17^\circ-i\cos 17^\circ}=-1.$$

47. Kub ildizni hisoblang:

1) $\sqrt[3]{1+i}$; | 2) $\sqrt[3]{-i}$; | 3) $\sqrt[3]{8}$; | 4) $\sqrt[3]{1-i}$; | 5) $\sqrt[3]{-8}$.

48. 4- darajali ildiz chiqaring:

1) $\sqrt[4]{-1}$; | 2) $\sqrt[4]{16}$; | 3) $\sqrt[4]{1+i}$; | 4) $\sqrt[4]{1-i}$; | 5) $\sqrt[4]{-16}$.

Nazorat ishi namunalari

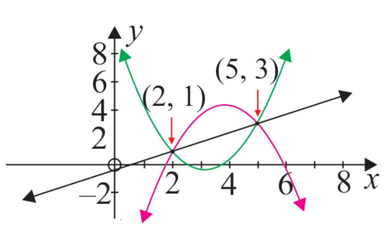
1. Hisoblang: $(35-7i)\cdot(4-6i)$.
2. Bo‘lishni bajaring: $\dfrac{8-i}{40+3i}$.
3. Ko‘paytiring:
   $3(\cos5°+i\sin5°)\cdot8(\cos3°+i\sin3°)$.
4. Darajaga ko‘taring: $(3(\cos4°+i\sin4°))^6$
5. Kvadrat ildiz chiqaring: $\sqrt{64i}$.

JAVOBLAR

III bob

73. a) Barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; b) ikkita nuqtada x abssissalar bir xil bo‘lgani uchun bu funksiya bo‘lmaydi; c) barcha nuqtalarda x abssissalar bir xil bo‘lgani uchun bu funksiya bo‘lmaydi; d) barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; e) barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; f) barcha nuqtalarda x abssissalar bir xil bo‘lgani uchun bu funksiya bo‘lmaydi. **74.** a) Funksiya; b) funksiya; c) funksiya; d) funksiya emas; e) funksiya; f) funksiya emas; g) funksiya; h) funksiya emas. **75.** Yo‘q, har qanday vertikal to‘g‘ri chiziq funksiya bo‘lmaydi. **76.** Yo‘q, $y=\pm\sqrt{9-x^2}$. **77.** a) 2; b) 8; c) –1; d) –13; e) 1. **78.** a) 2; b) 2; c) –16; d) –68; e) $\frac{32}{9}$ **79.** a) –3; b) 3; c) 3; d) –3; e) $\frac{15}{2}$. **80.** a) $7-3a$; b) $7+3a$; c) $-3a-2$; d) $10-3b$; e) $1-3x$; f) $7-3x-3h$. **81.** a) $2x^2+19x+43$; b) $2x^2-11x+13$; c) $2x^2-3x-1$; d) $2x^4+3x2-1$; e) $2x^4-x^2-2$; f) $2x^2+4hx+2h^2+3x+3h-1$. **82.** a) **I)** $-\frac{7}{2}$; **II)** $-\frac{3}{4}$; **III)** $-\frac{4}{9}$; b) $x=4$. **84.** $V(4)=6210$. Bu uskunaning 4 yildan so‘ng bo‘ladigan narxi. $t=4{,}5$ shuncha yildan keyin uskunaning narxi 5780 bo‘ladi. Uskunaning dastlabki narxi 9650 ga teng.

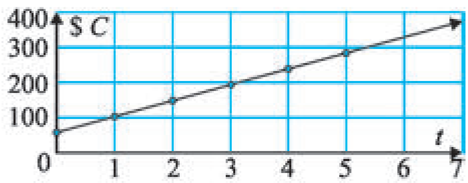
85.



86. $f(x)=-2x+5$. **87.** $a=3$, $b=-2$. **88.** $a=3$, $b=-1$, $c=-4$. **90.** a) **I)** $x>0$; b) **II)** $-2\le x\le3$; c) **I)** $-2<x\le0$; **II)** $0\le x<2$; d) **I)** $x\le2$; **II)** $x\ge2$; e) **II)** $x\in\mathbb{R}$; f) **I)** $x\in\mathbb{R}$; g) **I)** $1\le x\le5$; **II)** $x\le1$, $x\ge5$; h) **I)** $2\le x<4$, $x>4$; **II)** $x<0$, $0<x\le2$; i) **I)** $x\le0$, $2\le x\le6$; **II)** $0\le x\le2$, $x\ge6$. **92.** a) V(0)=25000 yevro. Bu avtomashinaning boshlang‘ich narxi; b) $V(3)=16\,000$. Bu avtomashinaning 3 yildan so‘ng bo‘lgan narxi; c) $t=5$.

93. a)

t	0	1	2	3	4	5
C	60	105	150	195	240	285

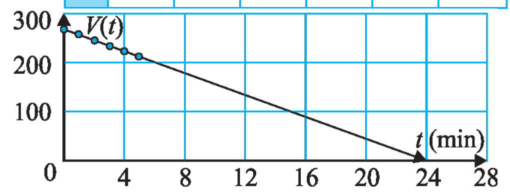


b) $C=60+45t$; c) $ 352,50.

94. a)

t	0	1	2	3	4	5
V	265	254	243	232	221	210

b)

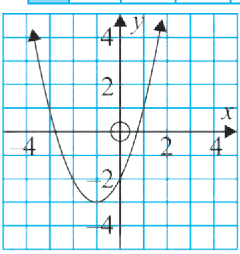


c) $V(t) = 265–11t$; d) **I)** 100 *l*.

95. a) Ha; b) yo‘q; c) ha; d) ha; e) ha; f) yo‘q. **96.** a) Yo‘q; b) ha; c) ha; d) ha; e) yo‘q; f) yo‘q. **97.** a) $x=-3$; b) $x=-2$ yoki –3; c) $x=1$ yoki 4; d) haqiqiy yechimga ega emas. **98.** a) **I)** 75 m; **II)** 195; **III)** 275 m; b) **I)** $t=2$ s yoki t=14 s; **II)** $t=0$ s yoki $t=16$ s. **99.** a) 40 ming, 480 ming; b) 10 ta yoki 62 ta.

100. a)

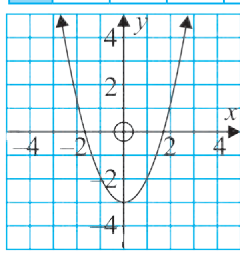
x	–3	–2	–1	0	1	2	3
y	1	–2	–3	–2	1	6	13



$y=x^2+2x-2$

b)

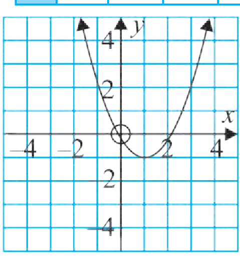
x	–3	–2	–1	0	1	2	3
y	6	1	–2	–3	–2	1	6



$y=x^2-3$

c)

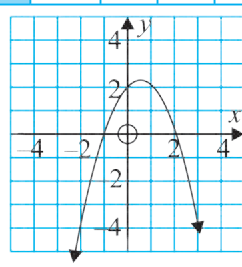
x	–3	–2	–1	0	1	2	3
y	15	8	3	0	–1	0	3



$y=x^2-2x$

d)

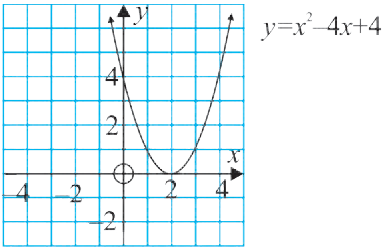
x	–3	–2	–1	0	1	2	3
y	–10	–4	0	2	2	0	–4



$f(x)=-x^2+x+2x$

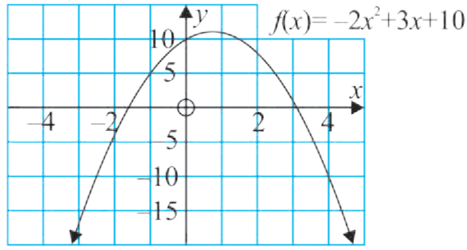
e)

x	–3	–2	–1	0	1	2	3
y	25	16	9	4	1	0	1



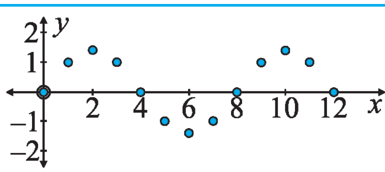
f)

x	–3	–2	–1	0	1	2	3
y	–17	–4	5	10	11	8	1

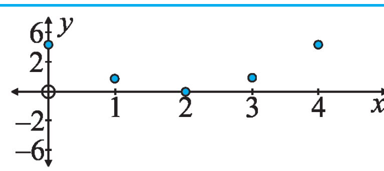


101. a) 3; b) –1; c) –4; d) 1; e) 5; f) 0; g) 8; h) –5; i) 2. **102.** a) 3; b) –6; c) 49; d) 15; e) 0; f) 20. **105.** a) x=3; b) x=–5/2; c) x=1; d) x=–4; e) x=3; f) x=–4. **106.** a) x=4; b) x=–2; c) x=1; d) x=11/2; e) x=5; f) x=–2. **107.** a) x=–3; b) x=4; c) x=–5/4; d) x=3/2; e) x=0; f) x=7/10; g) x=3; h) x=5/3; i) x=–4. **108.** a) (2, 3); b) (–1, 4); c) (3, 8); d) (0, 3); e) (–3, –18); f) (1, –1); g) (1/2, –5/4); h) (3/4, –7/8); i) (6, 7). **109.** a) $y=2(x-1)(x-2)$; b) $y=2(x-2)^2$; c) $y=(x-1)(x-3)$; d) $y=-(x-3)(x+1)$; e) $y=-3(x-1)^2$; f) $y=-2(x+2)(x-3)$. **110.** a) $y=3/2(x-2)(x-4)$; b) $y=-1/2(x+4)(x-2)$; c) $y=-4/3(x+3)^2$; d) $y=1/4(x+3)(x-5)$; e) $y=-(x+3)(x-3)$; f) $y=4(x-1)(x-3)$. **111.** a) 3m; b) 0,5s; c) 4m.

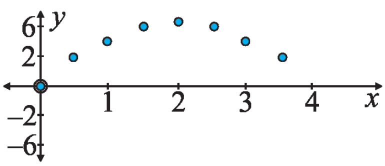
113. a) davriy;



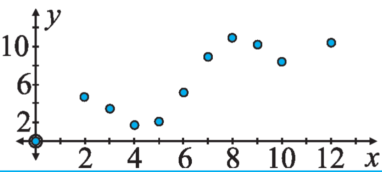
b) davriy emas;



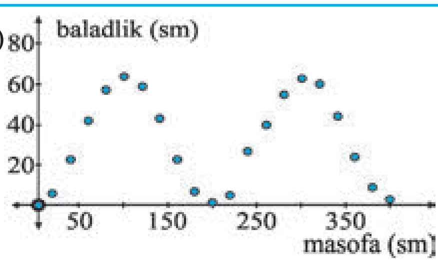
c) davriy emas;



d) davriy emas;



114. a)

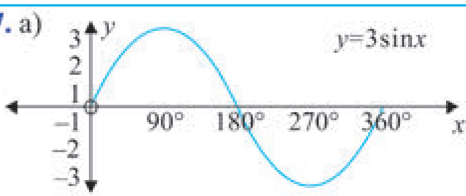


b) O‘q tenglamasi maksimum davr amplituda mos ravishda y=32; 64 sm; 200 sm; 32 sm ga teng.

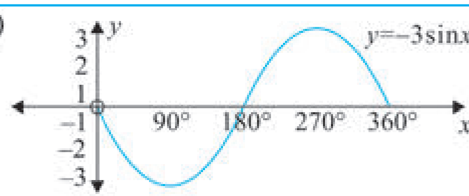
115. a) davriy; b) davriy; c) davriy; d) davriy emas; e) davriy; f) davriy.

116. a) 2; b) 8; c) (2,1); d) 8; e) y=–1.

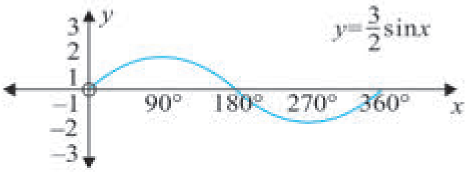
117. a)



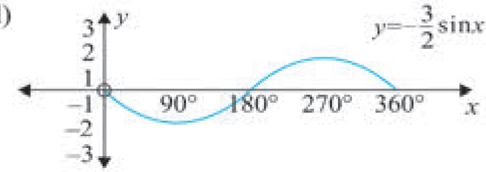
b)



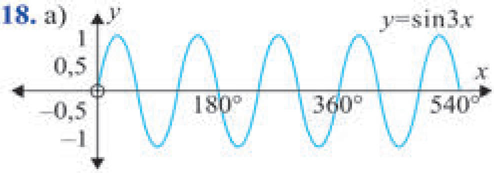
c)



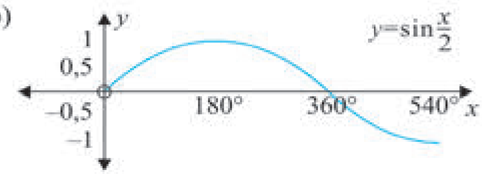
d)



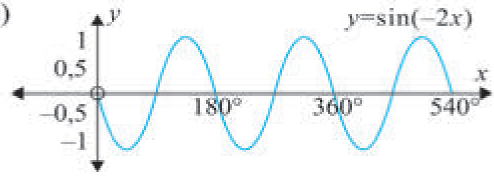
118. a)



b)



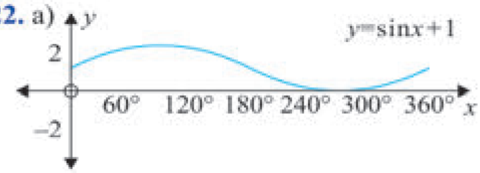
c)



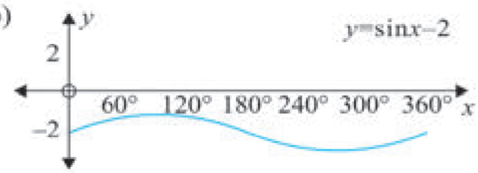
119. a) 90°; b) 90°; c) 1080°; d) 600°; **120.** a) $b=2/5$; b) $b=3$; c) $b=1/6$.

121. a) $y=3\sin x$; b) $y=\sin x-2$; c) $y=-2\sin x-1$; d) $y=\sin 2x$; e) $y=-4\sin(x/2)$; f) $y=\sin(x/2)$; g) $y=2\sin 3x$; h) $y=2\sin 2x-3$.

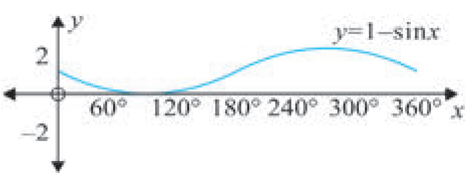
122. a)



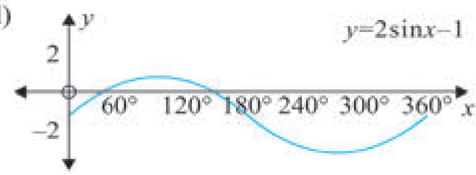
b)



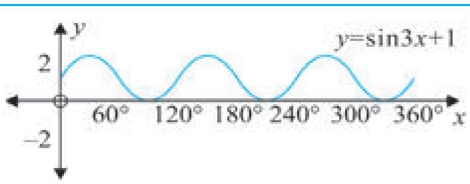
c)



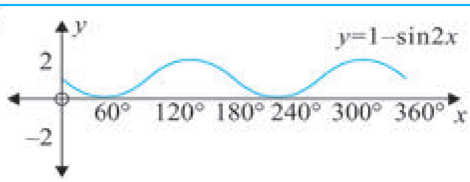
d)



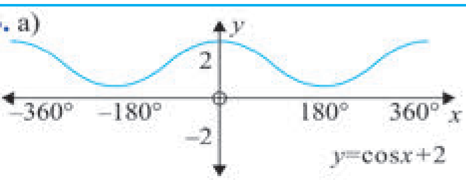
e)



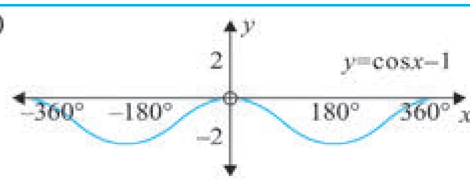
f)



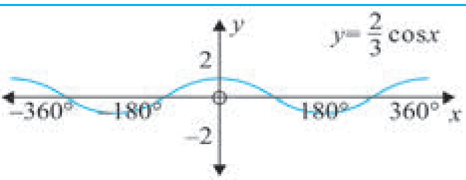
123. a)



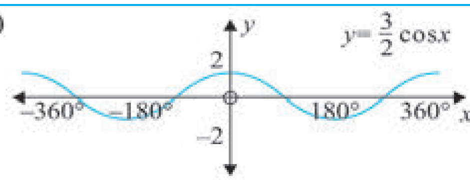
b)

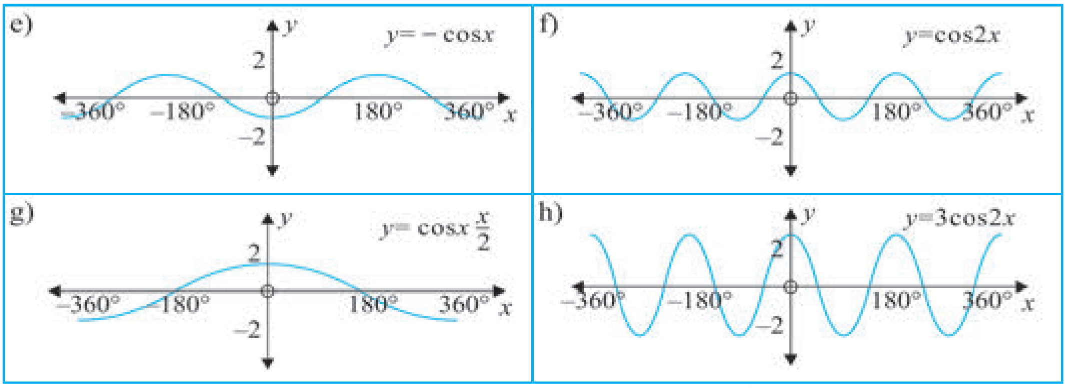


c)



d)





124. a) 120°; b) 1080°; c) 720°. **126.** a) y=2cos2x; b) y=cos(x/2)+2; c) y=–5$x$cos2x. **127.** T=9,5cos(30t)–9,5. **130.** 1) 0; 2) $\frac{\pi}{3}$; 3) $\frac{\pi}{6}$; 4) $-\frac{\pi}{3}$. **131.** 1) $-\frac{\pi}{4}$; 2) $-\frac{\pi}{6}$; 3) $\frac{\pi}{2}$; 4) $-\frac{\pi}{2}$. **132.** 1) $\frac{\pi}{2}$; 2) $\frac{5\pi}{6}$; 3) $\frac{\pi}{4}$; 4) π. **136.** 1) 0; 2) $\frac{4\pi}{3}$. **138.** 1) $\frac{3\pi}{2}$; 2) –π. **140.** 1) 2π; 2) $\frac{3\pi}{2}$.
142. 1) ma'noga ega; 2) ma'noga ega emas; 3) ma'noga ega emas.

144. 1) $x=(-1)^{n+1}\frac{\pi}{6}+n\pi,\ n\in Z.$ **146.** 1) $x=\pm\frac{3\pi}{4}+2n\pi,\ n\in Z.$

148. 1) $x=-\frac{\pi}{3}+n\pi,\ n\in Z.$ **150.** 1) $x=\pm\frac{2\pi}{3}+2n\pi,\ n\in Z.$

151. 1) $x=(-1)^{n}\frac{\pi}{4}+n\pi,\ n\in Z.$ **152.** 2) $x=-\frac{\pi}{24}+\frac{n\pi}{4},\ n\in Z.$

153. 1) $x_1=k\pi,\ x_2=\frac{\pi}{4}+k\pi,\ k\in Z.$

156. 1) $x_1=(-1)^{n+1}\frac{\pi}{4}+n\pi,\ x_2=-\frac{\pi}{8}+\frac{n\pi}{2},\ n\in Z.$

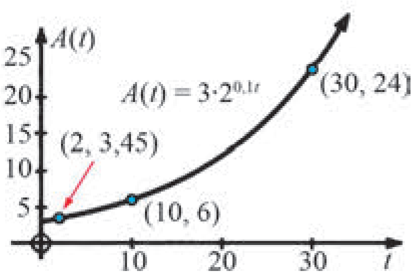
157. 1) $x_1=-\frac{\pi}{2}+2n\pi,\ \ x_2=(-1)^{n}\frac{\pi}{6}+n\pi,\ n\in Z.$

158. 2) $x=\pm\arccos(1-\frac{\sqrt{7}}{2})+2n\pi,\ n\in Z.$ **159.** 2) $x_1=-\frac{\pi}{4}+n\pi,$ $x_2=\arccos 4+n\pi,\ n\in Z.$ **160.** 1) $x=\frac{2n\pi}{3},\ n\in Z;$ 3) $x_1=\frac{n\pi}{2},$ $x_2=\frac{\pi}{10}+\frac{n\pi}{5},\ n\in Z.$ **162.** 1) $\left(\frac{\pi}{6};\ \frac{5\pi}{6}\right)$; 2) $\left(-\frac{\pi}{4};\ \frac{\pi}{4}\right)$; 3) $\left(-\frac{\pi}{3};\ \frac{\pi}{2}\right)$.

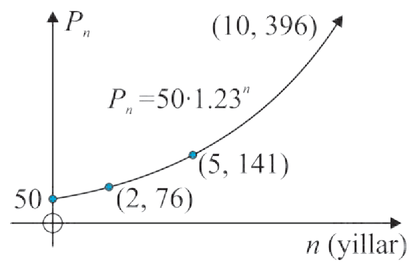
163. 1) $\left[\frac{\pi}{4}+2n\pi;\ \frac{3\pi}{4}+2n\pi\right],\ n\in Z;$ 2) $\left(\frac{3\pi}{4}+2n\pi;\ \frac{5\pi}{4}+2n\pi\right),\ n\in Z;$

3) $\left(-\frac{\pi}{2}+n\pi;\ -\frac{\pi}{4}+n\pi\right)$, $n \in Z$. **167.** 1) $\left[-\frac{\pi}{2}+n\pi;\ \frac{\pi}{3}+n\pi\right]$, $n \in Z$. **173.** 1) $y=2x+6$. **174.** 1) $y=13\cdot\sqrt{\frac{x-1}{17}}$. **175.** 1) $x^2+y^2=49$, aylana. **176.** 1) $(x-3)^2+(y-7)^2=36$, aylana. **177.** 1) 3; 2) 1; 3) 4; 4) 4. **178.** 1) katta; 2) kichik. **180.** 1) aniqlanish sohasi: $(-\infty;+\infty)$, qiymatlar sohasi: $(0;\ +\infty)$, $(-\infty;\ +\infty)$ oraliqda o‘sadi. **181.** 1) o‘sadi; 2) kamayadi; 3) o‘sadi. **183.** 1) $(-\infty;\ 1]$; 2) $\left(-\infty;\frac{4}{9}\right)$; 7) $[1;\ +\infty]$; 12) $\left(-\infty;\ -2-\sqrt{34}\right)\cup\left(-2+\sqrt{34};\ +\infty\right)$. **184.** 1) $(-\infty;\ 2]$. **185.** 1) 3; 2) –2; 3) –2; 4) –3; 5) –3. **186.** 1) katta; 2) katta; 3) kichik. **187.** 1) 2; 2) 5; 3) 125; 4) 45; 5) $\frac{1}{36}$; 9) –2. **188.** 1) $(2{,}5;\ +\infty)$; 2) $(-\infty;-1)\cup(3;+\infty)$; 3) $(-2;2)$. **190.** 1) $\frac{1}{32}$; 2) 1; 3) 4; 4) 2; 8) –2; 10) 0,5 va 1; 15) $\frac{1}{7}$ va 49. **191.** 1) $(64;\ +\infty)$; 2) $\left(0;\ \frac{1}{3}\right)\cup(27;+\infty)$; 7) (2;5).

192. a) 3 m^2;
b) **I)** 3,45 m^2; **II)** 6 m^2; **III)** 24 m^2;
c)

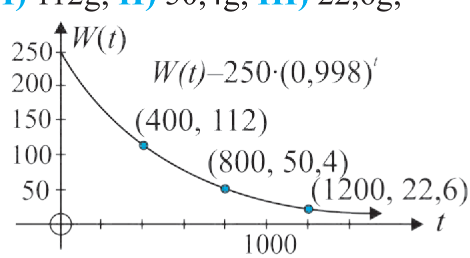


193. a) 50;
b) **I)** 76; **II)** 141; **III)** 396;
c)



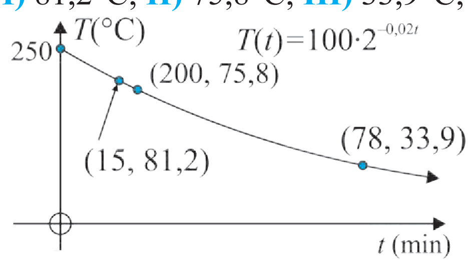
194. a) V_0; b) $2V_0$; c) 100%; d) 183 foizga ortadi.

195. a) 250g;
b) **I)** 112g; **II)** 50,4g; **III)** 22,6g;
c)

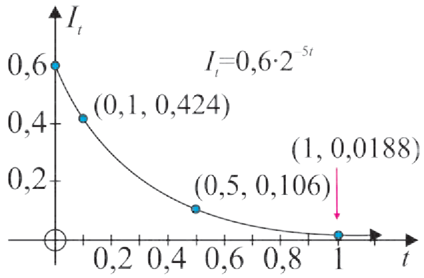


d) ≈346.

197. a) 100°C;
b) **I)** 81,2°C; **II)** 75,8°C; **III)** 33,9°C;
c)



198. a) 0,6 amper;
b) **I)** 0,424 amper; **II)** 0,106 amper; **III)** 0,0188 amper;
c)



199. a) L_0; b) 99%. **200.** Taxminan 3 soat 15 min. **201.** 37,8 oy. **202.** 10,8 minut. **203.** 22,7 yil. **204.** b) 1. **205.** a) $\{-14;\ 3;\ 15\}$; c) $\{-4;4\}$. **206.** a) $\left(-\infty;-\frac{1}{3}\right)\cup\left(\frac{3}{2};2\right)$.

207. a) $(-\infty;-6]\cup[-2;+\infty)$.

208. a) $\left(-2;\frac{-1-\sqrt{5}}{2}\right)\cup\left(\frac{-1+\sqrt{5}}{2};+\infty\right)$; c) $(-\infty;-2)\cup[1;+\infty)$.

209. a) $\{(7;8);(8;7);(-7;-8);+8;-7)\}$. **212.** a) 3; b) 2. **213.** a) kichik; b) kichik. **216.** a) $(-\infty;-1]\cup[1;+\infty)$; b) $(-\infty;+\infty)$. **217.** a) $(0;1]$; b) $(3;+\infty)$; c) $(-\infty;\ 0)$. **218.** a) $\frac{1}{15}$; b) 0 va 1; c) 1 va –2. **219.** c) 0. **220.** a) $\{(2;3);(-3;8)\}$. **221.** a) $(-\infty;0]$; b) $(-\infty;1,5)$. **222.** a) kichik; b) katta. **223.** a) $(-3,5;\ +\infty)$; b) $(-2;2)$. **224.** a) $2\sqrt{5}$. **225.** b) $(100000;0,1)$. **226.** a) $(3;1)$. **227.** a) $(-\infty;-2]\cup[1;+\infty)$. **229.** a) kichik; b) katta. **230.** a) $-\frac{2\pi}{3}$ **231.** c) $x_1=\frac{\pi}{4}+n\pi,\ x_2=\arccos\frac{1}{4}+n\pi,\ n\in Z.$

234. a) $\left(-\frac{\pi}{6}+2n\pi;\frac{7\pi}{6}+2n\pi\right),\ n\in Z.$ **235.** c) $\left(-\frac{\pi}{18}+\frac{\pi n}{9};\frac{\pi}{27}+\frac{n\pi}{9}\right),\ n\in Z.$

IV bob

1. 7) Re(z)=–7, Im(z)=3; 8) Re(z)=8, Im(z)=5; 9) Re(z)=–0,5, Im(z)=–6; 10) Re(z)=–5,7, Im(z)=–5; 11) Re(z)=0, Im(z)=–5; 12) Re(z)=90, Im(z)=0.

6. 1) $\bar{z}=7,2$; 3) $\bar{z}=4+3i$. **8.** 1) 16; 3) $3+i$. **10.** 1) $8i$; 2) $-1-5i$; 3) $-3+i$. **12.** 2) $1\frac{1}{6}-\frac{1}{6}i$. **14.** 1) $-\frac{23}{13}-\frac{2}{13}i$; 3) $\frac{5}{3}-\frac{2}{3}i$. **16.** 2) $\frac{12}{13}$. **20.** 1) 10; 2) 4; 3) 2; 4) 2. **22.** 1) 0; 2) $\frac{3\pi}{2}$; 3) $\frac{11\pi}{6}$. **24.** 1) $2\left(\cos\frac{7\pi}{4}+i\cdot\sin\frac{7\pi}{4}\right)$ va $2\cdot e^{\frac{7\pi i}{4}}$.

28. 1) $z_1\cdot z_2=\cos\frac{13\pi}{12}+i\cdot\sin\frac{13\pi}{12}$. **30.** 1) $\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$. **32.** 2) $\cos\frac{2\pi}{3}+i\cdot\sin\frac{2\pi}{3}$.

34. 1) $-\frac{42}{29}$; 2) $-18i$. **36.** 1) $z_0=\sqrt{2}(\cos\frac{\pi}{6}+i\cdot\sin\frac{\pi}{6})$, $z_1=\sqrt{2}(\cos\frac{7\pi}{6}+i\cdot\sin\frac{7\pi}{6})$.

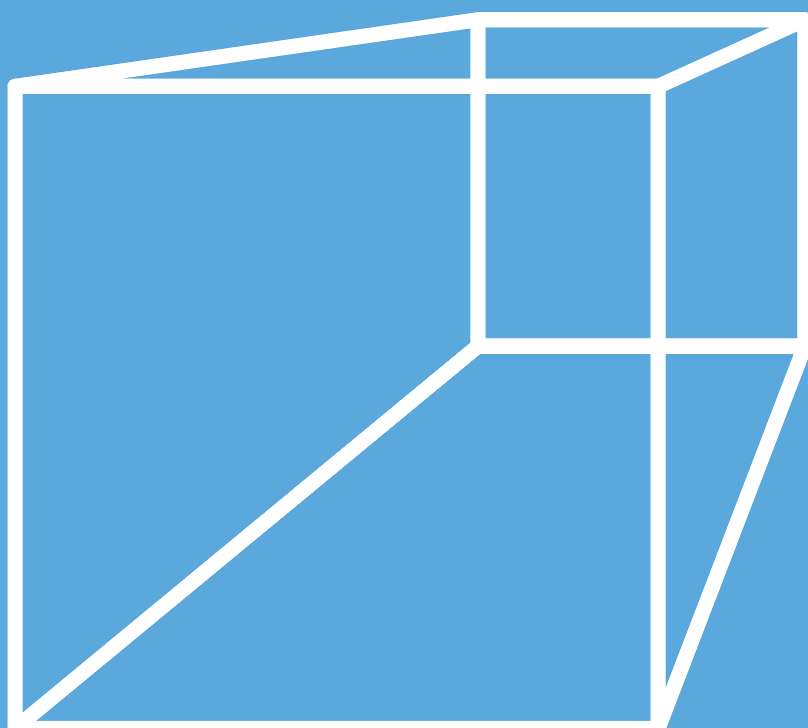
Foydalanilgan va tavsiya etiladigan adabiyotlar

1. *Sh.A. Alimov, O.R. Xolmuhamedov, M.A. Mirzaahmedov.* Algebra va analiz asoslari. 10- sinf uchun darslik. Toshkent: “O‘qituvchi”, 2004.
2. *Mal Coad and others.* Mathematics for the international students. Mathematical Studies SL 2nd edition. Haese and Harris publications. 2010.
3. *Э. Сайдаматов и др.* Алгебра и основы математического анализа.
часть 1, Ташкент: “O‘qituvchi”, 2016.
4. *A.U. Abduhamidov va boshqalar.* Algebra va matematik analiz asoslari, 1- qism, Toshkent: “O‘qituvchi”, 2012.
5. *Н.П. Филичева.* Уравнения и системы уравнений: Учебно-методическое пособие. “Рязань”. 2009.
6. *М.И. Исроилов.* Ҳисоблаш методлари. Тошкент: “Ўқитувчи” 1988.
7. *Г.К. Муравин.* Алгебра и начала анализа. Учебник для 10 класса. Москва, “Дрофа”, 2006.
8. Алгебра. Учебное пособие для 9–10 классов. Под ред. *Н.Я. Виленкина.* Москва, “Просвещение”, 2004.
9. http://www.ams.org/mathweb/ – Internetda matematika (ingliz tilida).
10. “Математика в школе” jurnali.
11. Fizika, matematika va informatika. Ilmiy-uslubiy jurnal (2001- yildan boshlab chiqa boshlagan).
12. *M.A. Mirzaahmedov, Sh.N. Ismailov* Matematikadan qiziqarli va olimpiada masalalari. I qism, Toshkent, “Turon-Iqbol”, 2016.
13. Matematikadan qo‘llanma, I va II qismlar. O‘qituvchilar uchun qo‘llanma. Prof. *T.A. Azlarov* tahriri ostida. Toshkent, “O‘qituvchi”, 1979.
14. *M.A. Mirzaahmedov, D.A. Sotiboldiyev.* O‘quvchilarni matematik olimpiadalarga tayyorlash. Toshkent, “O‘qituvchi”, 1993.
15. http://www.uzedu.uz – Xalq ta’limi vazirligining axborot ta’lim portali.
16. http://www.eduportal.uz – Multimedia markazi axborot ta’lim portali.
17. http://www.problems.ru – Matematikadan masalalar izlash tizimi (rus tilida).
18. http://matholymp.zn.uz – O‘zbekistonda va dunyoda matematik olimpiadalar.

MUNDARIJA

MATEMATIKA

GEOMETRIYA



10- sinf

10- sinfda geometriyaning stereometriya qismini – fazoviy geometrik shakllarning xossalarini tizimli o‘rganishga kirishiladi. Darslikdan asosiy fazoviy shakllar, ko‘pyoqlar va aylanma jismlar va ularning asosiy xossalari, fazoda parallel va perpendikular to‘g‘ri chiziqlar va tekisliklar hamda ularning xossalariga doir masalalar o‘rin olgan.

“Geometriya-10” darsligida nazariy materiallar sodda va ravon tilda ifoda etishga harakat qilingan. Barcha mavzu va tushunchalar turli hayotiy misollar orqali ochib berilgan. Har bir mavzudan so‘ng keltirilgan savollar, isbotlashga, hisoblashga va yasashga doir ko‘plab masala va misollar o‘quvchini ijodiy fikrlashga undaydi, o‘zlashtirilgan bilimlarni chuqurlashtirishga va mustahkamlab borishga yordam beradi.

“Geometriya-10” darsligi umumta’lim maktablarining 10- sinf o‘quvchilariga mo‘ljallangan, undan geometriyani mustaqil o‘rganmoqchi va takrorlamoqchi bo‘lgan kitobxonlar ham foydalanishlari mumkin.

MUNDARIJA

Darslikning “Geometriya” bo‘limida ishlatilgan belgilar va ularning talqini:

– teorema tavsifi
– aksioma tavsifi
– mavzu bo‘yicha savollar
– faollashtiruvchi mashg‘ulot
– teorema isboti oxiri
– amaliy tatbiq
– tarixiy lavhalar
– geometrik boshqotirmalar

IV BO‘LIM

FAZODA TO‘G‘RI CHIZIQLAR VA TEKISLIKLARNING PARALLELLIGI

10 FAZODA TO‘G‘RI CHIZIQLARNING O‘ZARO JOYLASHUVI

Fazodagi ikkita a va b to‘g‘ri chiziq bir tekislikda yotsa va kesishmasa, ular *parallel to‘g‘ri chiziqlar* deyiladi. a va b to‘g‘ri chiziqlarning parallelligi $a||b$ tarzda yoziladi.

Tekislikda berilgan nuqta orqali berilgan to‘gri chiziqqa yagona parallel to‘g‘ri chiziq o‘tkazish mumkin. Bunday xossa fazoda ham o‘rinli bo‘ladi:

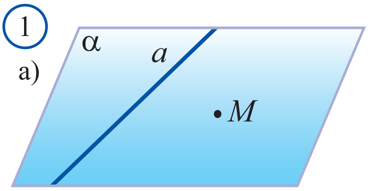
4.1- teorema. Fazoda berilgan to‘g‘ri chiziqda yotmagan nuqtadan shu to‘g‘ri chiziqqa yagona parallel to‘g‘ri chiziq o‘tkazish mumkin.

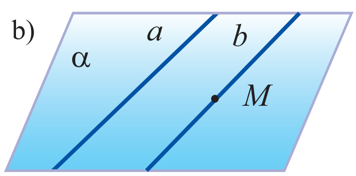
Isbot. a – berilgan to‘g‘ri chiziq va M – bu to‘g‘ri chiziqda yotmagan nuqta bo‘lsin (1.a- rasm). Isbotlangan 2.1- teoremaga ko‘ra, berilgan a to‘g‘ri chiziq va unda yotmagan M nuqta orqali yagona α tekislik o‘tkazish mumkin.

α tekislikda esa M nuqta orqali berilgan a to‘g‘ri chiziqqa parallel yagona b to‘g‘ri chiziqni o‘tkazish mumkin (1.b- rasm).

Xuddi shu b to‘g‘ri chiziq izlangan yagona to‘g‘ri chiziq bo‘ladi. ❑

1

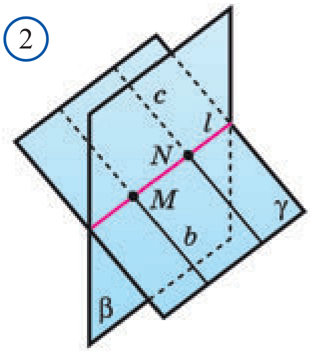




Tekislikda yotgan ikkita parallel to‘g‘ri chiziqdan biri uchinchi to‘g‘ri chiziqni kesib o‘tsa, ularning ikkinchisi ham bu to‘g‘ri chiziqni kesib o‘tadi. Shunga o‘xshash xossa fazoda ham o‘rinli bo‘ladi:

4.2- teorema. Fazoda berilgan ikkita parallel to‘g‘ri chiziqlardan biri tekislikni kesib o‘tsa, ularning ikkinchisi ham bu tekislikni kesib o‘tadi.

Isbot. b va c parallel to‘g‘ri chiziqlar berilgan bo‘lib, ularning biri – b to‘g‘ri chiziq berilgan β tekislikni M nuqtada kesib o‘tsin (2- rasm).



b va c to‘g‘ri chiziqlar parallel bo‘lgani uchun ular bitta tekislikda yotadi. Bu – γ tekislik bo‘lsin.

β va γ tekisliklar uchun M umumiy nuqta. Unda S3 aksiomaga ko‘ra, bu tekisliklar bitta l to‘g‘ri chiziq bo‘yicha kesishadi. Bu to‘g‘ri chiziq γ tekislikda yotadi va b to‘g‘ri chiziqni M nuqtada kesib o‘tadi. Shuning uchun, bu to‘g‘ri chiziq b to‘g‘ri chiziqqa parallel c to‘g‘ri chiziqni ham N nuqtada kesib o‘tadi.

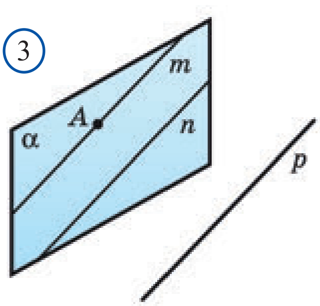
l to‘g‘ri chiziq β tekislikda ham yotgani uchun N nuqta bu β tekislikka ham tegishli bo‘ladi. Demak, N nuqta β va γ tekisliklar uchun umumiy nuqta.

Endi c to‘g‘ri chiziqning β tekislik bilan boshqa umumiy nuqtasi yo‘qligini ko‘rsatamiz. Teskarisini faraz qilamiz. Aytaylik, c to‘g‘ri chiziqning β tekislik bilan yana boshqa K umumiy nuqtasi bor bo‘lsin. Unda S2 aksiomaga ko‘ra, c to‘g‘ri chiziq β tekislikda yotadi. Unda c to‘g‘ri chiziq β va γ tekisliklar uchun umumiy bo‘ladi. Lekin l – bunday to‘g‘ri chiziq edi. Bundan c to‘g‘ri chiziqning l to‘g‘ri chiziq bilan ustma-ust tushishi kelib chqadi. Buning esa bo‘lishi mumkin emas. Chunki b to‘g‘ri chiziq c to‘g‘ri chiziqqa parallel va l to‘g‘ri chiziqni kesib o‘tadi. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi. ❑

Planimetriyadan sizga ma’lumki, ikki to‘gri chiqizning har biri uchinchi to‘g‘ri chiziqqa parallel bo‘lsa, ular o‘zaro parallel bo‘ladi. Bu xossa fazoda ham o‘rinli bo‘lib, u to‘g‘ri chiqizlarning parallellik alomati deb yuritiladi.

4.3- teorema. Uchinchi to‘g‘ri chiziqqa parallel ikki to‘gri chiziq o‘zaro parallеldir.

Isbot. Aytaylik, m va n to‘g‘ri chiziqlar p to‘g‘ri chiziqqa parallel bo‘lsin. m va n to‘g‘ri chiziqlarning bitta tekislikda yotishi va o‘zaro kesishmasligini, ya’ni parallel ekanligini ko‘ratamiz.



m to‘g‘ri chiziqda A nuqtani olamiz va bu nuqta va n to‘g‘ri chiziq orqali α tekislik o‘tkazamiz. m to‘g‘ri chiziqning α tekislikda yotishini isbotlaymiz.

Aytaylik, bunday bo‘lmasin. m to‘g‘ri chiziq α tekislik bilan umumiy nuqtaga ega bo‘lgani uchun, u tekislikni kesib o‘tadi. Unda 4.2- teoremaga ko‘ra, bu tekislikni m to‘g‘ri chiziqqa parallel bo‘lgan p to‘g‘ri chiziq ham, p to‘g‘ri chiziqqa parallel bo‘lgan n to‘g‘ri chiziq ham kesib o‘tadi. Lekin bunday bo‘lishi mumkin emas, chunki n to‘g‘ri chiziq α tekislikda yotadi.

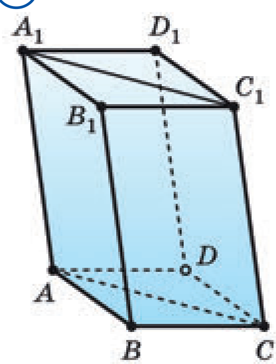
Demak, m va n to‘g‘ri chiziqlar α tekislikda yotadi.

Endi bu to‘gri chiziqlarning kesishmasligini isbotlaymiz. Yana teskarisini faraz qilamiz. m va n to‘g‘ri chiziqlar qandaydir B nuqtada kesishsin. Unda B nuqta orqali p to‘g‘ri chiziqqa parallel ikkita m va n to‘g‘ri chiziqlar o‘tadi. Buning esa, 4.1- teoremaga ko‘ra bo‘lishi mumkin emas.❑

Endi parallelepipedning quyidagi xossalarini isbotlaymiz.

1- xossa. ***$ABCDA_1B_1C_1D_1$ parallelepipedda (4- rasm) asos diagonallari va yon qirralardan tuzilgan ACC_1A_1 to‘rtburchak parallelogrammdan iborat bo‘ladi.***

4



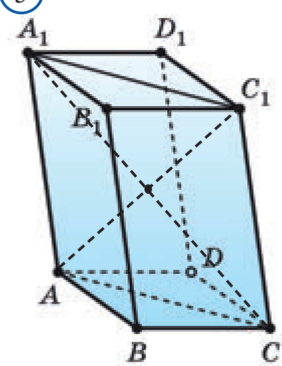
Haqiqatan, parallelepipedning ABB_1A_1 va BCC_1B_1 yoqlari ta’rifiga ko‘ra, parallelogrammdan iborat.

Bu parallelogrammlarning qarama-qarshi tomonlari o‘zaro teng bo‘ladi. Xususan, $AB = A_1B_1$ va $BC = B_1C_1$.

Parallelepiped ta’rifiga ko‘ra, $AA_1 \parallel BB_1$ va $BB_1 \parallel CC_1$. Unda 4.2- teoremaga ko‘ra, $AA_1 \parallel CC_1$ va $AA_1 = CC_1$ bo‘ladi. Demak, AC_1CA_1 to‘rtburchak – parallelogramm.

2- xossa. ***$ABCDA_1B_1C_1D_1$ parallelepipedning (4- rasm) qarama-qarshi yoqlari o‘zaro teng.***

5



Yuqoridagi xossaga ko‘ra, AC_1CA_1 – parallelogramm va $AC = A_1C_1$. Unda ABC va $A_1B_1C_1$ uchburchaklar uchta tomon bo‘yicha teng bo‘lib, ABC va $A_1B_1C_1$ burchaklar ham o‘zaro teng bo‘ladi. Natijada, $ABCD$ va $A_1B_1C_1D_1$ parallelogrammlar ham o‘zaro teng bo‘ladi.

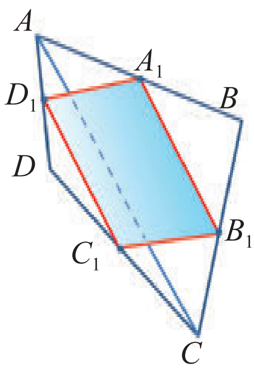
Boshqa qarama-qarshi yoqlarning tengligi ham shu tariqa isbotlanadi.

3- xossa. ***Parallelepipedning barcha diagonallari bitta nuqtada kesishadi va bu nuqtada teng ikkiga bo‘linadi (5- rasm).***

1- xossaga ko‘ra, AC_1CA_1 – parallelogramm. Unda bu parallelogrammning diagonallari A_1C va AC_1 bitta nuqtada kesishadi va kesishish nuqtasida teng ikkiga bo‘linadi.

Qolgan diagonallarning kesishishi va bu nuqtada teng ikkiga bo‘linishi shunga o‘xshash isbotlanadi.

Bitta to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi kesmalar



(nurlar) o‘zaro *parallel kesmalar* (*nurlar*) deb ataladi.

Masala. Uchlari bitta tekislikda yotmaydigan fazoviy to‘rtburchak tomonlarining o‘rtalari parallelogrammning uchlari bo‘lishini isbotlang.

Isbot. $ABCD$ – fazoviy to‘rtburchak va A_1, B_1, C_1 va D_1 – to‘rtburchak tomonlarining o‘rtalari bo‘lsin (6- rasm). U holda, A_1B_1 kesma – ABC uchburchakning AC omoniga parallel o‘rta chizig‘i, C_1D_1 esa ACD uchburchakning AC tomoniga parallel o‘rta chizig‘i bo‘ladi.

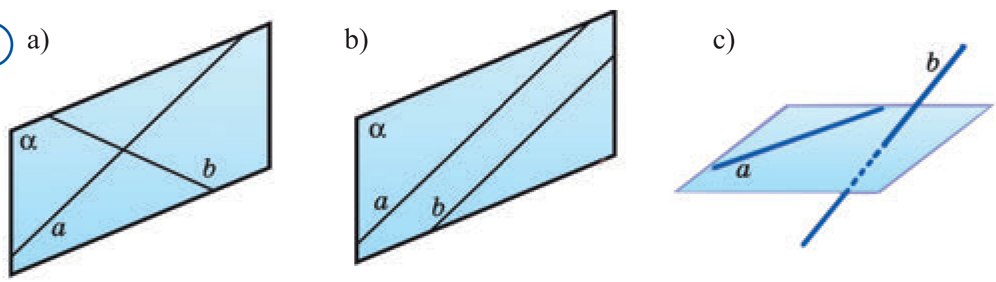
4.3- teoremaga ko‘ra, A_1B_1 va C_1D_1 to‘g‘ri chiziqlar parallel bo‘ladi. Demak, ular bir tekislikda yotadi.

A_1D_1 va B_1C_1 to‘g‘ri chiziqlarning parallelligi ham xuddi shunday isbotlanadi.

Shunday qilib, $A_1B_1C_1D_1$ to‘rtburchak bitta tekislikda yotadi va uning qarama-qarshi tomonlari parallel. Demak, u parallelogrammdir.❑

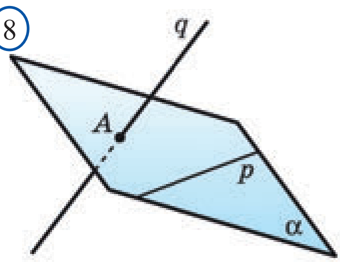
Agar fazoda ikki to‘g‘ri chiziq o‘zaro kesishsa yoki o‘zaro parallel bo‘lsa, ular bitta tekislikda yotadi (7.a va 7.b- rasm). Fazoda bitta tekislikda yotmaydigan to‘g‘ri chiziqlar *ayqash to‘g‘ri chiziqlar* deb ataladi (7.c- rasm).

Ayqash to‘g‘ri chiziqlarni quyidagi alomatiga ko‘ra tanib olish mumkin:

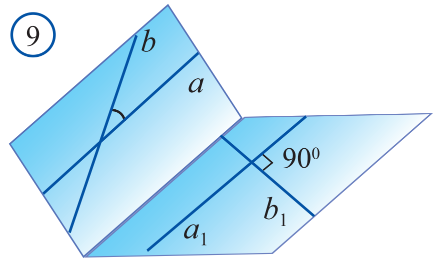


4.4- teorema. Agar ikki to‘g‘ri chiziqdan biri biror tekislikda yotsa, ikkinchisi esa bu tekislikni birinchi to‘g‘ri chiziqda yotmagan nuqtada kesib o‘tsa, u holda bu to‘g‘ri chiziqlar ayqash bo‘ladi.

Isbot. Aytaylik, p to‘g‘ri chiziq α tekislikda yotsin. q to‘g‘ri chiziq esa bu tekislikni p to‘g‘ri chiziqqa tegishli bo‘lmagan A nuqtada kesib o‘tsin (8- rasm). p va q to‘g‘ri chiziqlarning ayqash ekanligini isbotlatmiz.

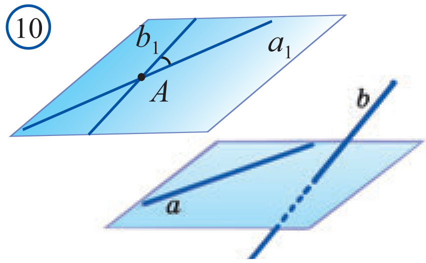


Teskarisini faraz qilamiz: p va q to‘g‘ri chiziqlar birorta β tekislikda yotsin. U holda β tekislikka p to‘g‘ri chiziq va A nuqta tegishli bo‘ladi. O‘z navbatida A nuqta q tekislikka ham tegishli. Demak, α va β tekisliklar ustma-ust tushadi. Natijada, shartga ko‘ra a tekislikka tegishli bo‘lmagan q to‘g‘ri chiziq bu tekislikka tegishli bo‘lib qoldi. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi. ❑



Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan qo‘shni burchaklarning kichigi *ikki to‘g‘ri chiziq orasidagi burchak* deyiladi.

Ayqash to‘gri chiziqlar orasidagi burchak deb, bu to‘g‘ri chiziqlarga parallel bo‘lgan kesishuvchi to‘g‘ri chiziqlar orasidagi burchakka aytiladi (9- rasm).



Amalda a va b ayqash to‘g‘ri chiziqlar orasidagi burchakni topish uchun (10- rasm)

1) biror A nuqta tanlanadi;

2) A nuqtadan ayqash to‘g‘ri chiziqlarga parallel a_1 va b_1 to‘g‘ri chiziqlar o‘tkaziladi;

3) bu to‘g‘ri chiziqlar orasidagi burchak o‘lchanadi.

Bu algoritm natijasi A nuqtaga bog‘liq emasligi haqida o‘ylab ko‘ring.

Orasidagi burchak 90° ga teng to‘g‘ri chiziqlar *perpendikular to‘g‘ri chiziqlar* deb ataladi. Parallel to‘g‘ri chiziqlar orasidagi burchak 0° ga deb hisoblanadi.

? *Mavzuga doir savollar va mashqlar*

1. *Parallel to‘g‘ri chiziqlarning qanday xossalarini bilasiz?*
2. *To‘g‘ri chiziqlarning parallellik alomatini ayting*
3. *Parallelepipedning qanday xossalarini bilasiz?*
4. *To‘g‘ri chiziqlarning ayqashlik alomatini ayting.*
5. *To‘g‘ri chiziqlar orasidagi burchak qanday aniqlanadi?*
6. *Ayqash to‘gri chiziqlar parallel bo‘lishi mumkinmi?*

4.1. a) $ABCDA_1B_1C_1D_1$ parallelepipeddagi; b) $ABCA_1B_1C_1$ prizmadagi parallel qirralar juftlarini aniqlang.

4.2. Qanday piramidalarda parallel qirralar bo‘ladi?

4.3. Ma’lumki, tekislikda to‘g‘ri chiziq parallel to‘g‘ri chiziqlardan birini kesib o‘tsa, ikkinchisini ham kesib o‘tadi. Bu xossa fazoda ham o‘rinli bo‘ladimi?

4.4. To‘g‘ri tasdiqni toping:

a) fazoda to‘g‘ri chiziqda yotmagan nuqtadan unga parallel ko‘plab to‘g‘ri chiziqlar o‘tkazish mumkin;

b) uchinchi to‘g‘ri chiziqqa parallel to‘g‘ri chiziqlar o‘zaro kesishadi; c) agar ikki to‘g‘ri chiziq tekislikda yotsa, ular kesishadi; d) to‘g‘ri chiziqdan va unda yotmagan nuqtadan ikkita turli tekislik o‘tkazish mumkin; e) fazoning tekislikda yotmagan nuqtasidan bu tekislikni kesadigan ko‘plab to‘g‘ri chiziqlar o‘tkazish mumkin.

4.5. *A* uchi α tekislikda yotgan *AB* kesmada *C* nuqta tanlangan. *B* va *C* nuqtalardan o‘tkazilgan parallel to‘g‘ri chiziqlar α tekislikni, mos ravishda, B_1 va C_1 nuqtalarda kesib o‘tadi. Agar: a) *C* nuqta *B* kesmaning o‘rtasi, va $BB_1 = 14$ sm; b) $AC:CB = 3:2$ va $BB_1 = 50$ sm bo‘lsa, CC_1 kesmaning uzunligini toping.

4.6. Bitta tekislikda yotmaydigan *MNOP* parallelogramm va *EK* asosli *MNEK* trapetsiya berilgan. a) *PO* va *EK* to‘g‘ri chiziqlarning o‘zaro joylashishini aniqlang; b) trapetsiyaning asoslari $MN = 45$ sm, $EK = 55$ sm ga teng bo‘lib, unga ichki aylana chizish mumkin. Trapetsiyaning perimetrini toping.

4.7. *a* va *b* to‘g‘ri chiziqlar bitta tekislikda yotadi. Bu to‘g‘ri chiziqlarning mumkin bo‘lgan o‘zaro joylashishini ko‘rsating.

A) *a* va *b* parallel; *B*) *a* va *b* kesishadi; *C*) *a* va *b* kesishmaydi; *D*) *a* va *b* ayqash; *E*) *a* va *b* parallel emas.

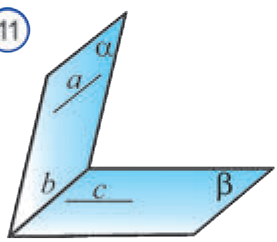
4.8. *a* va *b* to‘g‘ri chiziqlar *c* to‘gri chiziqqa parallel. *a* va *b* to‘g‘ri chiziqlar o‘zaro qanday joylashishi mumkin?

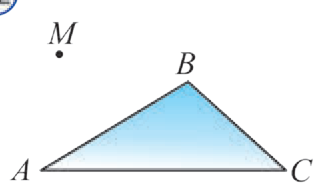
4.9. 11- rasmda α va β tekisliklar *b* to‘g‘ri chiziq bo‘ylab kesishadi. Agar $a||b$, *c* va *b* to‘g‘ri chiziqlar parallel bo‘lmsa, *a* va *c* to‘g‘ri chiziqlar o‘zaro qanday joylashishi mumkin?

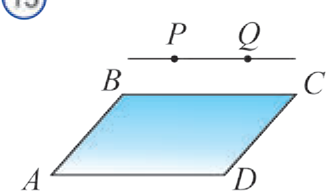
4.10. 12- rasmda *M* nuqta *ABC* uchburchakning tashqi sohasida yotibdi. *MA, MC, MB* to‘g‘ri chiziqlarga ayqash to‘gri chiziqlarni aniqlang.

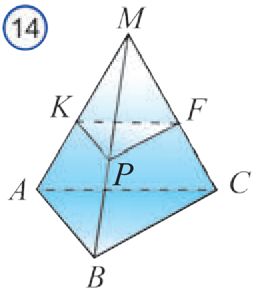
4.11. 13- rasmda *PQ* to‘g‘ri chiziq *ABCD* to‘rtburchakning tashqi sohasida yotadi va *BC* ga parallel. a) *PQ* va *AB*; b) *PQ* va *CD*; c) *PQ* va *AD* qanday to‘g‘ri chiziqlar?

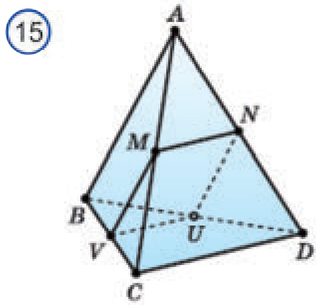
4.12. 14- rasmda *M* nuqta *ABC* uchburchakning tashqi sohasida yotibdi. *MA, MB, MC* kesmalarning o‘rtalari, mos ravishda, *K, F, P* nuqtalar bilan belgilangan. 1) *KP*; 2) *PF*; 3) *KF*; 4) *KM*; 5) *PM*; 6) *FM*; 7) *AB*; 8) *BC*; 9) *AC* to‘g‘ri chiziqlardan qaysilari o‘zaro parallel?

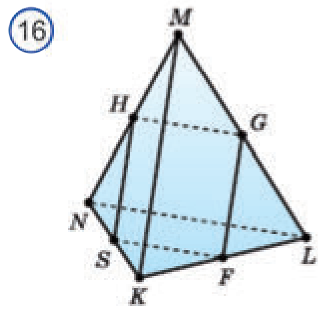
11


12


13








4.13. *M, N, U, V* nuqtalar *ABCD* piramidaning, mos ravishda, *AC, AD, BD* va *BC* qirralarining o‘rtalari (15- rasm). Agar *AB* = 20 sm, *CD* = 30 sm bo‘lsa, *MNUV* to‘rtburchakning perimetrini toping.

4.14. *H, G, F, S* nuqtalar uchburchakli *KLMN* piramidaning, mos ravishda, *MN, ML, LK* va *KN* qirralarining o‘rtalari (16- rasm). Agar *LK* = 18 mm, *MN* = 22 mm bo‘lsa, *HGFS* to‘rtburchakning perimetrini toping.

4.15. To‘g‘ri chiziqdan turli ikkita tekislik o‘tkazish mumkinligini isbotlang.

4.16. Bitta tekislikda yotmagan to‘rtta nuqta berilgan. Ularning uchtasi orqali nechta tekislik o‘tkazish mumkin?

4.17. *A, B, C* nuqtalar berilgan ikkita tekislikning har birida yotadi. Bu nuqtalarning bitta tekislikda yotishini isbotlang.

4.18. *a* to‘g‘ri chiziq bo‘ylab kesishuvchi ikkita tekislik berilgan. *b* to‘g‘ri chiziq ulardan birida yotadi va ikkinchisini kesib o‘tadi. *a* va *b* to‘g‘ri chiziqlarning kesishishini isbotlang.

4.19. Uchta tekislikning har ikkitasi o‘zaro kesishadi. Tekisliklarning kesishish to‘g‘ri chiziqlaridan ikkitasi biror nuqtada kesishsa, uchunchi kesishish chizig‘i ham bu nuqtadan o‘tishini isbotlang.

4.20. Agar to‘rtburchakning diagonallari kesishsa, unda uning uchlari bitta tekislikda yotishini isbotlang.

4.21. *K, Z, M, N* nuqtalar *SABC* uchburchakli piramidaning, mos ravishda, *SA, AC, BC*, *SB* kesmalarining o‘rtalari. Agar piramidaning yon qirralari *b*, asosining tomoni *a* ga teng bo‘lsa, *KZMN* to‘rtburchakning perimetrini toping.

4.22. *XU* va *VT* to‘g‘ri chiziqlar parallel, *XY* va *VT* to‘g‘ri chiziqlar esa ayqash. Agar: a) $\angle YXU = 40^\circ$; b) $\angle YXU = 135^\circ$; c) $\angle YXU = 90^\circ$ bo‘lsa, *XY* va *VT* to‘g‘ri chiziqlar orasidagi burchakni toping.

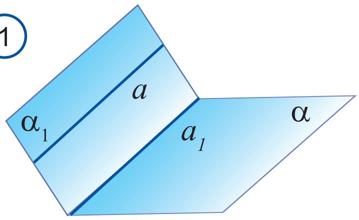
4.23. *l* to‘g‘ri chiziq *ABCD* parallelogrammning *BC* tomoniga parallel va uning tekisligida yotmaydi. *l* va *CD* to‘g‘ri chiziqlar ayqash ekanligini isbotlang. Agar piramidaning burchaklaridan biri: a) 58°; b) 133° bo‘lsa, *l* va *CD* to‘g‘ri chiziqlar orasidagi burchakni toping.

11 FAZODA TO‘G‘RI CHIZIQLAR VA TEKISLIKNING O‘ZARO JOYLASHUVI

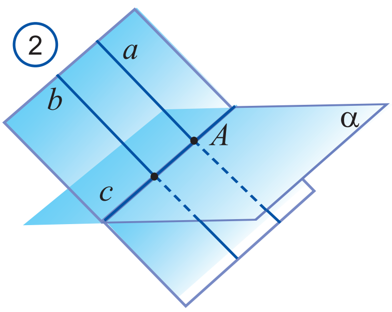
Agar to‘g‘ri chiziq bilan tekislik kesishmasa, *to‘g‘ri chiziq va tekislik parallel* deyiladi. To‘g‘ri chiziq bilan tekislikning paralleligi quyidagi alomat orqali aniqlanadi.

4.5- Teorema. Agar tekislikda yotmagan to‘g‘ri chiziq shu tekislikdagi biror to‘g‘ri chiziqqa parallel bo‘lsa, bu to‘g‘ri chiziq tekislikning o‘ziga ham parallel bo‘ladi.

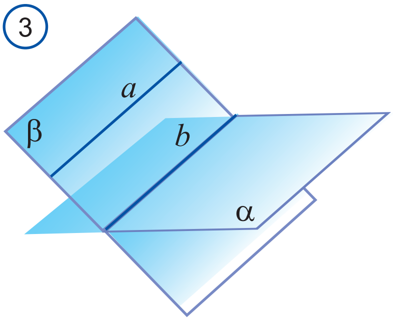
1



2



3



Isbot. Aytaylik, α – tekislik, a – unda yotmagan to‘g‘ri chiziq, a_1 esa α tekislikda yotgan va a ga parallel to‘g‘ri chiziq bo‘lsin.

a va a_1 to‘g‘ri chiziqlar orqali α_1 tekislikni o‘tkazamiz (1- rasm). Ravshanki, α va α_1 tekisliklar a_1 to‘g‘ri chiziq bo‘yicha kesishadi.

Agar a to‘g‘ri chiziq α tekislikni kesib o‘tsa, u holda kesishish nuqtasi a_1 to‘g‘ri chiziqqa tegishli bo‘lar edi. Ammo buning iloji yo‘q, chunki a va a_1 to‘g‘ri chiziqlar o‘zaro parallel. Shunday qilib, a to‘g‘ri chiziq α tekislikni kesib o‘ta olmaydi.

Demak, a to‘g‘ri chiziq α tekislikka parallel. ❑

Masala. Agar tekislik ikki parallel to‘g‘ri chiziqdan birini kesib o‘tsa, ikkinchisini ham kesib o‘tishini isbotlang.

Isbot. a va b – ikki parallel to‘g‘ri chiziq, α esa a to‘g‘ri chiziqni A nuqtada kesib o‘tuvchi tekislik bo‘lsin (2- rasm).

a va b to‘g‘ri chiziqlardan tekislik o‘tkazamiz. U α tekislikni biror c to‘g‘ri chiziq bo‘yicha kesadi. c to‘g‘ri chiziq a to‘g‘ri chiziqni A nuqtada kesib o‘tadi.

Demak, unga parallel bo‘lgan b to‘g‘ri chiziqni ham kesib o‘tadi. c to‘g‘ri chiziq α tekislikda yotgani uchun α tekislik b to‘g‘ri chiziqni ham kesib o‘tadi.

4.6- teorema. Agar bir tekislik ikkinchi tekislikka parallel bo‘lgan to‘g‘ri chiziqdan o‘tsa, bu tekisliklarning kesishish to‘g‘ri chizig‘i ham berilgan to‘g‘ri chiziqqa parallel bo‘ladi.

Isbot. Aytaylik, a to‘g‘ri chiziq α tekislikka parallel va β tekislikda yotsin.

b to‘g‘ri chiziq esa α va β tekisliklarning kesishish chizig‘i bo‘lsin (3- rasm). U holda, a va b to‘g‘ri chiziqlar β tekislikda yotadi va o‘zaro kesishmaydi. Aks holda, a to‘g‘ri chiziq β tekislikni kesib o‘tar edi.

Demak, a va b to‘g‘ri chiziqlar o‘zaro parallel.❑

? *Mavzuga doir savollar va mashqlar*

1. *To‘g‘ri chiziq va tekislik fazoda o‘zaro qanday joylashishi mumkiin?*
2. *To‘g‘ri chiziq va tekislik qachon parallel bo‘ladi?*
3. *To‘g‘ri chiziqning tekislikka parallellik alomatini ayting.*
4. *Fazoda to‘g‘ri chiziq va tekisliklarning joylashuvi bilan bog‘liq qanday xossalarni bilasiz?*

4.24. a) $ABCDA_1B_1C_1D_1$ kubning; b) $ABCDEFA_1B_1C_1D_1E_1F_1$ oltiburchakli muntazam prizmaning bir-biriga parallel bo‘lgan qirra va yoqlarini aniqlang.

4.25. To‘g‘ri tasdiqni tanlang:

a) fazoda to‘g‘ri chiziqda yotmagan nuqtadan bu to‘g‘ri chiziqqa parallel ko‘plab to‘gri chiziqlar o‘tkazish mumkin;

b) uchinchi to‘g‘ri chiziqqa parallel to‘g‘ri chiziqlar bitta nuqtada kesishadi;

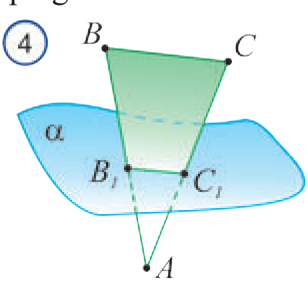
c) agar to‘g‘ri chiziqning ikki nuqtasi tekislikka tegishli bo‘lsa, to‘gri chiziq tekislikni kesib o‘tadi;

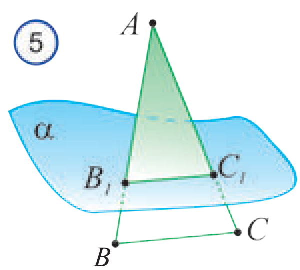
d) to‘g‘ri chiziq va unda yotmagan nuqtadan ikkita har xil tekislik o‘tkazish mumkin;

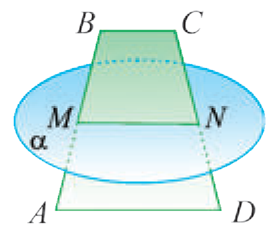
e) fazoda tekislikda yotmagan nuqtadan berilgan tekislikni kesib o‘tuvchi ko‘plab to‘g‘ri chiziqlar o‘tkazish mumkin.

4.26. A va C nuqtalar α tekislikda yotadi. B va D nuqtalar β tekislikda yotadi. AC, CD, BD, AB, BC va AD to‘g‘ri chiziqlardan qaysilari β tekislikni kesib o‘tadi?

4.27. ABC uchburchak α tekislikni B_1 va C_1 nuqtalarda kesib o‘tadi (4- rasm). Agar $AB_1 : BB_1 = 2 : 3$, $BC = 15$ sm, $BC || B_1C_1$ bo‘lsa, B_1C_1 kesmaning uzunligini toping.

4


5


6


4.28. α tekislik ABC uchburchakning AB va AC tomonlarini B_1 va C_1uqtalarda kesib o‘tadi (5- rasm). Agar $AB_1 : BB_1 = 3 : 1$, $B_1C_1 = 12$ sm, $BC \| \alpha$ bo‘lsa, BC kesmaning uzunligini toping.

4.29. α tekislik $ABCD$ trapetsiyaning AD asosiga parallel va yon tomonlarini M va N nuqtalarda kesib o‘tadi (6- rasm). Agar $AD = 17$ sm, $BC = 9$ sm bo‘lsa, MN kesmaning uzunligini toping.

4.30. Tekislikka unda yotmagan nuqtadan nechta parallel to‘g‘ri chiziq o‘tkazish mumkin?

4.31. a to‘g‘ri chiziq α tekislikka parallel. To‘g‘ri tasdiqlarni toping.

a) a to‘g‘ri chiziq α tekislikning faqat bitta to‘g‘ri chizig‘iga parallel bo‘ladi;

b) a to‘g‘ri chiziq α tekislikning bitta to‘g‘ri chizig‘idan boshqa barcha to‘g‘ri chiziqlariga ayqash bo‘ladi;

c) α *t*ekislikkda a to‘g‘ri chiziqqa parallel va ayqash bo‘lgan ko‘plab to‘g‘ri chiziqlar topiladi;

d) α *t*ekislikkda faqat bitta a to‘g‘ri chiziqqa parallel va bu tekislikning ixtiyoriy nuqtasidan o‘tuvchi to‘g‘ri chiziq mavjud.

4.32. A, B, C, D nuqtalar bitta tekislikda yotmaydi. M, N, K, Z nuqtalar, mos ravishda, AD, BD, BC, AC kesmalarning o‘rtalari.Agar $CD=AB$ bo‘lsa, MK va NZ to‘g‘ri chiziqlarning perpendikularligini isbotlang.

4.33. $ABCD$ parallelogrammning AB va BC tomonlari α tekislikni kesib o‘tadi. AD va DC to‘g‘ri chiziqlar ham α tekislini kesib o‘tishini isbotlang.

4.34. ABC va ABD uchburchaklar bitta tekislikda yotmaydi. CD to‘g‘ri chiziqqa parallel bo‘lgan ixtiyoriy to‘g‘ri chiziqning bu uchburchaklar tekisligini kesib o‘tishini isbotlang.

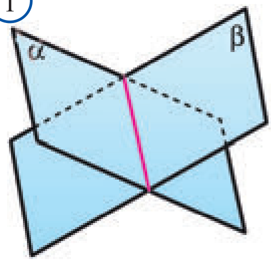
4.35. Berilgan ikki to‘g‘ri chiziqni kesib o‘tuvchi to‘g‘ri chiziqlarning bir tekislikda yotishini isbotlang.

12 FAZODA TEKISLIKLARNING O‘ZARO JOYLASHUVI

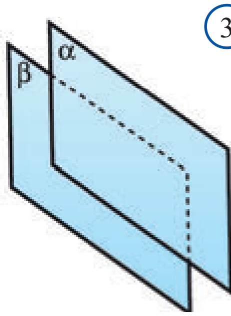
Ikki to‘g‘ri chiziq yoki umumiy nuqtaga ega, yoki umumiy nuqtaga ega bo‘lmasligi mumkin. Birinchi holda S3 aksiomaga ko‘ra bu tekisliklar umumiy to‘g‘ri chiziqqa ham ega bo‘ladi, ya’ni to‘g‘ri chiziq bo‘ylab kesishadi (1- rasm). Ikkinchi holda tekisliklar kesishmaydi (2- rasm).

Kesishmaydigan tekisliklar *parallel tekisliklar* deb ataladi. Parallel tekisliklar haqida xonaning poli va shifti, qarama-qarshi devorlari tasavvur berishi mumkin (3- rasm).

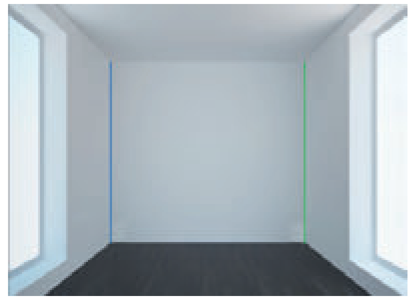
1



2



3

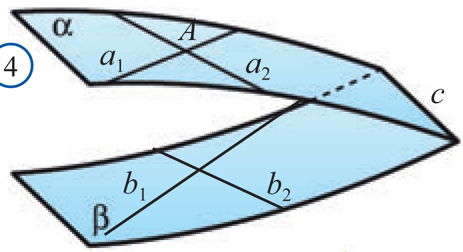


Ikki tekislikning paralleligi quyidagi alomat orqali aniqlanadi.

4.7- teorema. Agar bir tekislikdagi kesishuvchi ikki to‘g‘ri chiziq ikkinchi tekislikdagi ikki to‘g‘ri chiziqqa mos ravishda parallel bo‘lsa, bu tekisliklar parallel bo‘ladi.

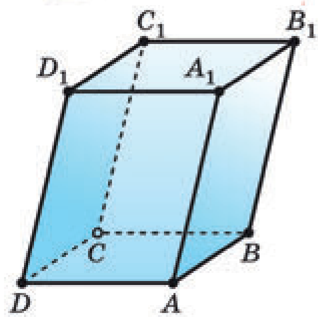
Isbot. Aytaylik, α va β – berilgan tekisliklar, a va b – α tekislikda yotgan va A nuqtada kesishuvchi to‘g‘ri chiziqlar, a_1 va b_1 esa β tekislikda yotgan va, mos ravishda, a va b to‘g‘ri chiziqlarga parallel to‘g‘ri chiziqlar bo‘lsin (4- rasm).

4



Faraz qilamiz, α va β tekisliklar o‘zaro parallel bo‘lmasin, ya’ni qandaydir c to‘g‘ri chiziq bo‘ylab kesishsin. U holda 4.6- teoremaga ko‘ra, a_1 va a_2 to‘g‘ri chiziqlar, mos ravishda, b_1 va b_2 to‘g‘ri chiziqlarga parallel bo‘lib, β tekislikka ham parllel bo‘ladi. Shuning uchun ular bu tekislikda yotgan c to‘g‘ri chiziqni ham kesib o‘tmaydi.

5



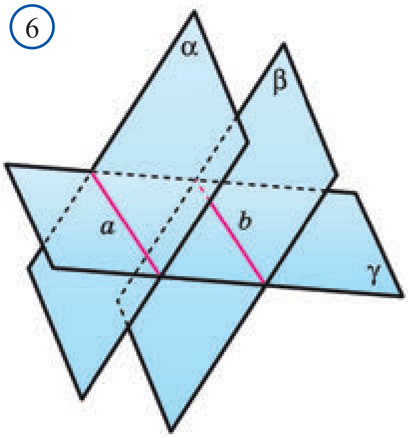
Shunday qilib, α tekislikda yotgan A nuqta orqali c to‘g‘ri chiziqqa parallel ikkita a_1 va a_2 to‘g‘ri chiziq o‘tmoqda. Parallellik aksiomasiga ko‘ra, bunday bo‘lishi mumkin emas. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi. ❑

Bu teoremadan foyadalanib, parallelepipedning yon yoqlari (5- rasm) parallel bo‘lishini mustaqil isbotlang.

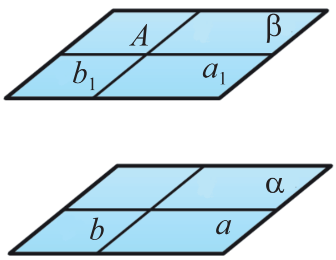
4.8- teorema. Ikki parallel tekslikning uchinchi tekislik bilan kesishish to‘g‘ri chiziqlari o‘zaro parallel bo‘ladi.

Isbot. Aytaylik, α va β parallel tekisliklar γ tekislikni, mos ravishda, a va b to‘g‘ri chiziqlar bo‘ylab kesib o‘tsin (6- rasm). a va b to‘g‘ri chiziqlar parallel ekanligini isbotlaymiz.

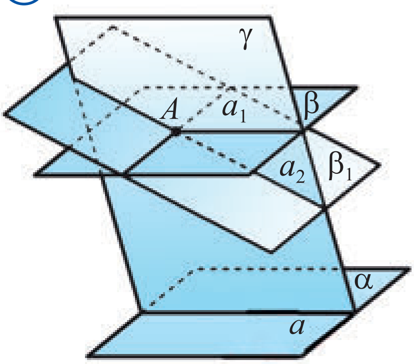
6



7



8



Faraz qilamiz, a va b to‘g‘ri chiziqlar biror Q nuqtada kesishsin. U holda Q nuqta α tekislikda yotadi, chunki a to‘g‘ri chiziq α tekislikda yotadi. Shuningdek, Q nuqta β tekislikda yotadi, chunki b to‘g‘ri chiziq β tekislikda yotadi. Natijada, α va β tekisliklar umumiy Q nuqtaga ega bo‘lmoqda. Buning esa, shartga ko‘ra, iloji yo‘q. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi. ❑

4.9- teorema. Berilgan tekislikka undan tashqaridagi nuqtadan yagona parallel tekislik o‘tkazish mumkin.

Isbot. Berigan α tekislikda kesishadigan ikkita a, b to‘g‘ri chiziq o‘tkazamiz. Berilgan A nuqtadan ularga parallel a_1, b_1 to‘g‘ri chiziqlarni o‘tkazamiz (7- rasm).

a_1, b_1 to‘g‘ri chiziqlar orqali β tekislik o‘tkazamiz. Bu tekislik 4.7- teoremaga ko‘ra, α tekislikka parallel bo‘lib, izlanayotgan tekislik bo‘ladi.

Endi bu tekislikning yagonaligini ko‘rsatamiz. Faraz qilamiz, α tekislikka parallel yana bitta $β_1$ tekislik mavjud bo‘lsin (8- rasm). A nuqtadan va a to‘g‘ri chiziqdan o‘tuvchi γ tekislikni o‘tkazamiz. Bu tekislik β tekislikni a_1 to‘g‘ri chiziq bo‘ylab, $β_1$ tekislikni a_2 to‘g‘ri chiziq bo‘ylab kesib o‘tadi. a_1, a_2 to‘g‘ri chiziqlar 4.6- teoremaga ko‘ra a to‘g‘ri chiziqqa parallel bo‘ladi. Lekin buning bo‘lishi mumkin emas, chunki tekislikda unda yotmagan nuqtadan faqat bitta parallel to‘g‘ri chiziq o‘tkazish mumkin. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi. ❑

4.10- teorema. Uchinchi tekislikka parallel ikki tekislik o‘zaro parallel bo‘ladi.

Bu teoremani mustaqil isbotlang.

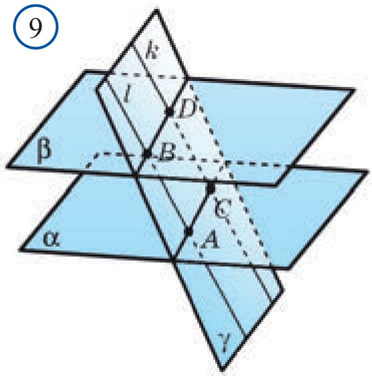
4.11- teorema. Parallel tekisliklar orasidagi parallel to‘g‘ri chiziqlar kesmalari tengdir.

Isbot. Aytaylik, α va β tekisliklar k va l to‘g‘ri chiziqlardan AB va CD kesmalarni ajratsin (9- rasm).

Bu kesmalarning tengligini ko‘rsatamiz.

k va l to‘g‘ri chiziqlardan o‘tuvchi γ tekislik parallel tekisliklarni AC va BD to‘g‘ri chiziqlar bo‘ylab kesib o‘tadi. Natijada, qarama-qarshi tomonlari parallel bo‘lgan $ABCD$ to‘rtburchakka, ya’ni parallelogrammga ega bo‘lamiz. Parallelogrammning qarama-qarshi tomonlari o‘zaro teng bo‘ladi. Xususan, $AB = CD$. ❑

9



4.11- teorema. Uchta parallel tekisliklar orasidagi ixtiyoriy to‘g‘ri chiziqlar kesmalari o‘zaro proporsional bo‘ladi.

Teoremani mustaqil isbotlang.

Mavzuga doir savollar va mashqlar

1. *Tekisliklar fazoda qanday joylashishi mumkiin?*
2. *Parallel tekisliklar deb qanday tekisliklarga aytiladi?*
3. *Tekisliklarning parallellik alomatini ayting.*
4. *Fazoda tekisliklarning joylashuvi bilan bog‘liq qanday xossalarni bilasiz?*
5. *Parallelepipedning yon yoqlari parallel bo‘lishini asoslang.*

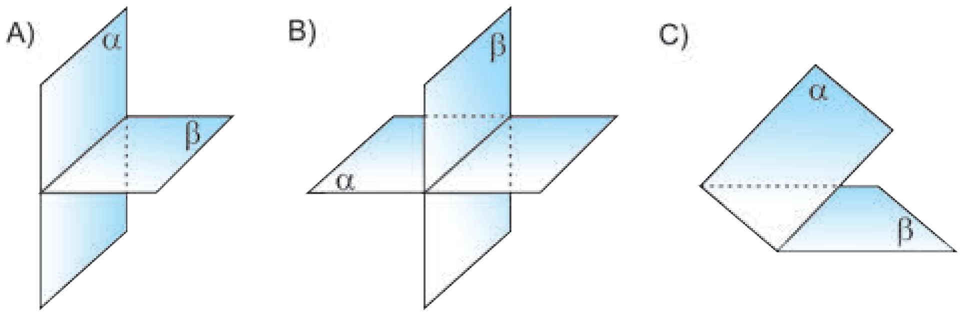
4.36. a) $ABCDA_1B_1C_1D_1$ parallelepipedning; b) $ABCA_1B_1C_1$ prizmaning parallel yoqlarini aniqlang.

4.37. Birorta ham umumiy nuqtasi bo‘lmagan α va β tekisliklar fazoda qanday joylashadi?

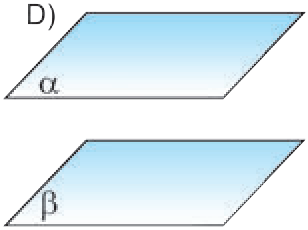
4.38. α va β tekisliklar parallel. a va b to‘g‘ri chiziqlar α tekislikkda yotadi, c va d to‘g‘ri chiziqlar esa β tekislikda yotadi. Quyidagi tasdiqlardan qaysilari to‘g‘ri:

1) $a \parallel b$; 2) $c \parallel b$; 3) $b \parallel b$; 4) $b \parallel a$; 5) $c \parallel a$; 6) $d \parallel b$; 7) $a \parallel a$; 8) $d \parallel a$.

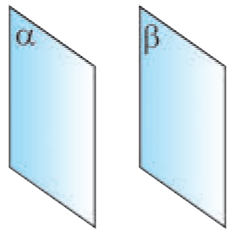
10



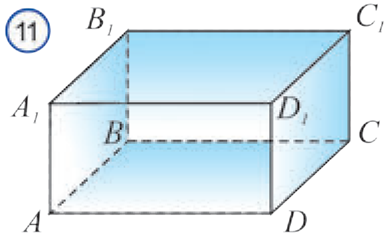
D)



E)



11



4.39. Kesishuvchi ikkita tekislik tasvirlangan uchta rasmni ko‘rsating (10- rasm).

4.40. α va β tekisliklar parallel. Ularning hech biriga tegishli bo‘lmagan nuqtadan γ tekislik o‘tkazilgan. To‘g‘ri tasdiqlarni ko‘rsating.

a) γ tekislik α tekislikka parallel bo‘lgan yagona tekislik;

b) γ tekislik β tekislikni kesib o‘tuvchi yagona tekislik;

c) γ tekislik β tekislikka parallel bo‘lgan yagona tekislik;

d) γ tekislik α tekislikni kesib o‘tuvchi yagona tekislik;

e) γ tekislik α tekislikka ham, β tekislikka ham parallel bo‘lgan yagona tekislik.

4.41. 11- rasmda $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepiped tasvirlangan.

a) $A_1B_1C_1D_1$ va B_1A_1AB; b) ADD_1A_1 va $ABCD$; c) ABB_1A_1 va C_1D_1DC; d) $BADC$ va ABB_1A_1; e) CC_1B_1B va ADD_1A_1 tekisliklarning o‘zaro joylashuvini aniqlang.

4.42. *AB, BC* kesmalar *ABCD* parallelogrammning tomonlari bo‘lib, ular, mos ravishda, *a* va *b* to‘g‘ri chiziqlarga parallel (12- rasm). *a* va *b* to‘g‘ri chiziqlar o‘zaro kesishadi va α tekislikka tegishli. *ABCD* va α tekisliklarning fazoda o‘zaro joylashuvini aniqlang.

4.43. *a* va *b* ayqash to‘g‘ri chiziqlar berilgan. *a* to‘g‘ri chiziqdan o‘tuvchi va β tekislikka parallel bo‘lgan nechta tekislik o‘tkazish mumkin?

4.44. Ikkita α va β tekisliklarning kesishish chizig‘i uchinchi γ tekislikka parallel. α va β tekisliklarning fazoda o‘zaro joylashuvini aniqlang.

4.45. *AB* va *CD* parallel to‘g‘ri chiziqlar orqali o‘tkazilgan γ tekislik α va β parallel tekisliklarni, mos ravishda, *AC* va *BD* to‘g‘ri chiziqlar bo‘ylab kesib o‘tadi (13- rasm). Agar $BD = 15$ sm bo‘la, *AC* kesma uzunligini toping.

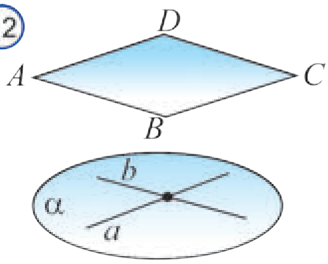
4.46. Ixtiyoriy ikkita ayqash to‘g‘ri chiziq orqali yagona parallel tekisliklar juftini o‘tkazish mumkinligini isbotlang.

4.47. α va β tekisliklar parallel. α tekislikda yotuvchi ixtiyoriy to‘g‘ri chiziq β tekislikka parallel bo‘lishini isbotlang.

4.48. *O* nuqta – bir tekislikda yotmaydigan AA_1, BB_1, CC_1 kesmalarning umumiy o‘rtasi. *ABC* va $A_1B_1C_1$ tekisliklar parallel ekanligini isbotlang.

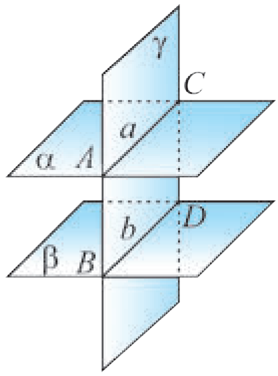
4.49. *ABCD* parallelogramm va uni kesmaydigan tekislik berilgan. Parallelogramning *A, B, C, D* uchlaridan tekislikni, mos ravishda, A_1, B_1, C_1, D_1 nuqtalarda kesib o‘tadigan parallel to‘g‘ri chiziqlar o‘tkazilgan. Agar AA_1 = 4 m, BB_1 = 3 m va CC_1 = = 1 m bo‘lsa, DD_1 kesma uzunligini toping.

12



4.50. Ikkita parallel tekislik berilgan. Bir tekislikning *A* va *B* nuqtalaridan ikkinchi tekislikni A_1 va B_1 nuqtalarda kesib o‘tuvchi parallel to‘gri chiziqlar o‘tkazilgan. Agar *AB* = *a* bo‘lsa, A_1B_1 kesma uzunligini toping.

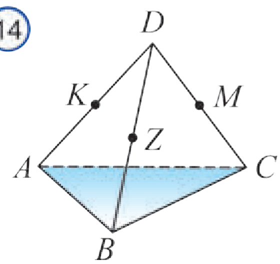
13



4.51. α va β tekisliklar parallel. α tekislikning *M* va *N* nuqtalaridan β tekislikni *K* va *L* nuqtalarda kesib o‘tuvchi parallel to‘gri chiziqlar o‘tkazilgan. *MNLK* parallelogramm ekanligini isbotlang. Agar *ML* = 14 sm, *NK* = 8 sm va *MK : MN* = 9 : 7 bo‘lsa, *MNLK* to‘rtburchak perimetrini toping.

4.52. *OF* va *OP* nurlar α va β parallel tekisliklarni, mos ravishda, F_1, P_1, F_2, P_2 nuqtalarda kesib o‘tadi. Agar F_1P_1 = 3 sm, F_2P_2 = 5 sm va P_1P_2= 4 sm bo‘lsa, OP_1 kesma uzunligini toping.

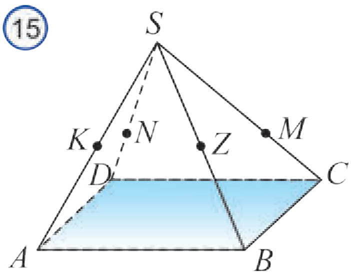
14



4.53. *OA* va *OB* nurlar α va β parallel tekisliklarni, mos ravishda, A_1, B_1, A_2, B_2 nuqtalarda kesib o‘tadi. Agar OA_1 = 16 sm, A_1A_2 = 24 sm va A_2B_2= 50 sm bo‘lsa, A_1B_1 kesma uzunligini toping.

4.54. *D* nuqta *ABC* uchburchak tekisligiga tegishli emas (14- rasm). *K, M, Z* nuqtalar, mos ravishda, *DA, DB* va *DC* kesmalarning o‘rtasi. *ABC* va *KZM* tekisliklarning o‘zaro joylashuvini aniqlang.

15



4.55. *S* nuqta *ABCD* parallelogramm tekisligiga tegishli emas (15- rasm). *K, Z, M, N* nuqtalar, mos ravishda, *SA, SB, SC* va *SD* kesmalarga tegishli. Agar *SK = AK, SZ = BZ, SM : MC* = 2 : 1, *SN : ND* = 2 : 1 bo‘lsa, *ABCD* va *KZMN* tekisliklarning o‘zaro joylashuvini aniqlang.

13 FAZODA PARALLEL PROYEKSIYA

Fazodagi shakllar turli usullar bilan tekislikda tasvirlanadi. Quyda ular bilan tanishamiz.

Fazodagi shaklni tekislikka *parallel proyeksiyalash* deb shunday akslantirishga aytiladiki, unda shaklning har bir nuqtasi berilgan proyeksiyalash yo‘nalishiga parallel bo‘lgan to‘g‘ri chiziqlar bo‘ylab tekislikka ko‘chiriladi.

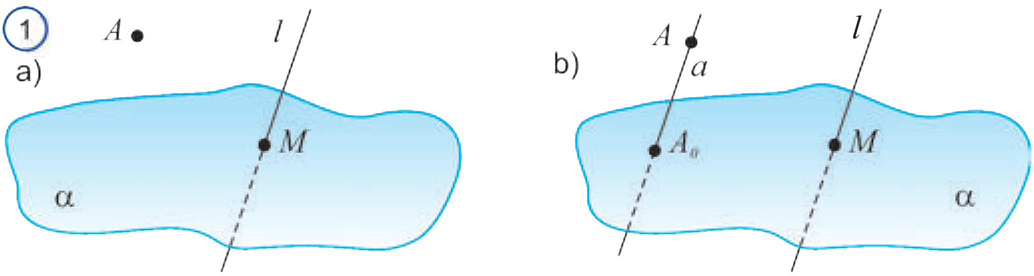
Parallel proyeksiyalashni yorug‘lik nurlari yordamida biror narsaning devor yoki polga tushirilgan soyasiga qiyoslash mumkin.

Shunday qilib, parallel proyeksiyalashda biror shakl va *proyeksiyalash tekisligi* deb nomlanuvchi tekislik olinadi hamda *proyeksiyalash yo‘nalishi*, ya’ni biror to‘g‘ri chiziq tanlanadi. Albatta, bu to‘g‘ri chiziq proyeksiya tekisligi bilan kesishishi lozim.

Aytaylik, ixtiyoriy α tekislik va proyeksiyalash to‘g‘ri chizig‘i l va tekislikda ham, to‘g‘ri chiziqda ham yotmagan A nuqta berilgan bo‘lsin (1.a- rasm).

A nuqtadan α tekislikka l to‘g‘ri chiziqqa parallel bo‘lgan a to‘g‘ri chiziq o‘tkazamiz. Bu to‘g‘ri chiziq α tekiislikni A_0 nuqtada kesib o‘tsin (1.b- rasm).

1



Topilgan A_0 nuqta A nuqtaning α tekislikka *parallel proyeksiyasi* deb ataladi.

Aytaylik, biror F shaklni α tekislikka l yonalish bo‘yicha parallel proyeksiyalash lozim bo‘lsin. Buning uchun F shaklning ixtiyoriy nuqtasini olamiz, undan l ga parallel to‘g‘ri chiziq o‘tkazamiz va uning α tekislik bilan kesishish nuqtasini belgilaymiz. Bunday nuqtalar α tekislikda qandaydir F_1 shaklni hosil qiladi. Aynan shu F_1 shakl F shakilning α tekislikdagi parallel proyeksiyasi bo‘ladi. 2- rasmda F shaklning α tekislikka proyeksiyasi – F_1 shakl tasvirlangan.

Parallel proyeksiyalashning quyidagi xossalarini keltirib o‘tamiz. Ularni mustaqil isbotlab ko‘ring.

Parallel proyeksiyalashda: nuqta – nuqtaga, kesma – kesmaga, to‘g‘ri chiziq– to‘g‘ri chiziqqa o‘tadi.

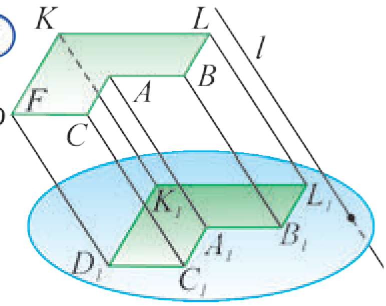
Parallel to‘g‘ri chiziqlar proyeksiyalari parallel bo‘ladi yoki ustma-ust tushadi.

Quyidagi xossalarni isbotlaylik.

1- xossa. ***Shaklning to‘g‘ri chiziqli kesmlari proyeksiyasi ham kesmalardan iborat bo‘ladi.***

Haqiqatan, AC kesmaning nuqtalarini proyeksiyalovchi barcha to‘g‘ri chiziqlar α tekislikni A_1C_1 to‘g‘ri chiziq bo‘yicha kesib o‘tuvchi tekislikda yotadi (3- rasm). AC kesmaning ixtiyoriy B nuqtasi A_1C_1 kesmaning B_1 nuqtasiga o‘tadi.❑

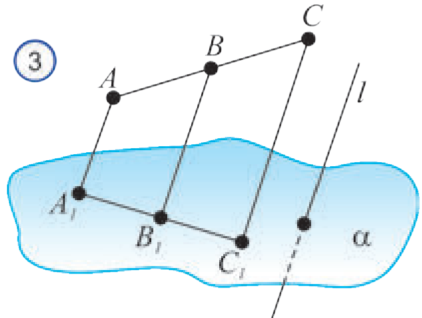
2



2- xossa. ***Shaklning parallel kesmlari proyeksiyasi ham parallel kesmalardan iborat bo‘ladi.***

Haqiqatan, AC va BD biror shaklning parallel kesmalari bo‘lsin (4- rasm). Ularning proyeksiyalari – A_1C_1 va B_1D_1 kesmalar ham parallel bo‘ladi, chunki ularni ikki parallel tekislikni α tekislik bilar kesganda hosil qildik.

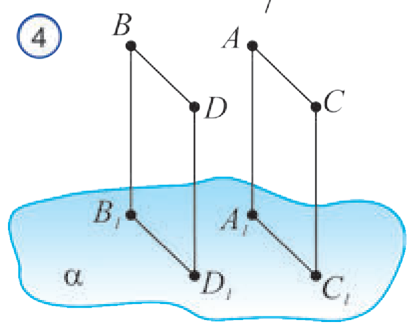
3



3- xossa. ***Bitta to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotgan kesmalar uzunliklari nisbati o‘z proeksiyalari uzunliklari nisbatiga teng.***

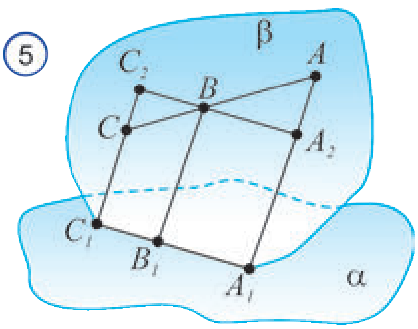
Haqiqatan, 5- rasmda AC va A_1C_1 to‘g‘ri chiziqlar β tekislikda yotadi. AC kesmaning B nuqtasidan A_1C_1 ga parallel bo‘lgan A_2C_2 to‘g‘ri chiziqni o‘tkazamiz.

4



Hosil bo‘lgan BAA_2 va BCC_2 uchburchaklar o‘xshash bo‘ladi. Uchburchaklarning o‘xshashligi va $A_1B_1=A_2B$ va $B_1C_1=BC_2$ tengliklardan izlanayotgan nisbatga ega bo‘lamiz: $AB:BC=$ $=A_1B_1:B_1C_1$. ❑

5



Shunday qilib, parallel proyeksilashda to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotgan kesmalar uzunliklari nisbati saqlanar ekan.

Xususan, kesmaning o‘rtasi proyeksiya o‘rtasiga o‘tadi.

? *Mavzuga doir savollar va mashqlar*

1. *Fazodagi shaklni tekislikka parallel proyeksiyalash deb qanday akslantirishga aytiladi?*

2. Nuqtaning tekislikka parallel proyeksiyasi qanday topiladi?
3. Parallel proyeksiyalash tekisligi va proyeksiyalash yo'nalishi deb nimaga aytiladi?
4. Parallel proyeksiyalashning qanday xossalarini bilasiz?
5. Parallel proyeksiyalashdan qayerda foydalanish mumkin?

4.56. Parallel proyeksiyalashda kesmaning proyeksiyasi: a) kesma; b) nuqta; c) ikki nuqta; d) nur; e) to'g'ri chiziq bo'lishi mumkinmi?

4.57. Parallel proyeksiyalashda kvadratning proyeksiyasi: a) kvadrat; b) parallelogramm; c) romb; d) to'g'ri to'rtburchak; e) trapetsiya; f) kesma bo'lishi mumkinmi?

4.58. Parallel tekisliklardan birida yotgan uchburchak ikkinchi tekislikka parallel proyeksiyalansa, uning yuzi o'zgarmasligini isbotlang.

4.59. Parallelogrammning parallel proyeksiyasi trapetsiya bo'lishi mumkinmi? Javobingizni asoslang.

4.60. Muntazam uchburchakning parallel proyeksiyasi mutazam uchburchak bo'ladimi?

4.61. To'g'ri burchakli uchburchakning parallel proyeksiyasi to'g'ri burchakli uchburchak bo'ladimi?

4.62. ABC uchburchakning parallel proyeksiyasi $A_1B_1C_1$ uchburchakdan iborat. Bu proyeksiyalashda ABC ucburchakning: a) medianasi; b) balandligi; c) bissektrisasi. $A_1B_1C_1$ uchburchakning mos: a) medianasi; b) balandligi; c) bissektrisasiga o'tadimi?

4.63. ABC uchburchakning parallel proyeksiyasi $A_1B_1C_1$ uchburchakdan iborat. Agar $\angle A = 30°$, $BC = 20$ sm bo'lsa, $\angle A_1 = 30°$, $B_1C_1 = 20$ sm bo'ladimi?

4.64. AB kesmaning parallel proyeksiyasi A_1B_1 kesmadan iborat. AB kesmadan olingan C nuqtaning proyeksiyasi esa C_1 nuqta. $AB = 48$ sm, $A_1B_1 = 36$ sm. Agar AC kesmaning uzunligi: a) 24 sm; b) 12 sm; c) 8 sm; d) 32 sm; e) 36 sm bo'lsa, A_1C_1 kesmaning uzunligini toping.

14 AMALIY MASHQ VA TATBIQ

4.65. a) Ikki to'g'ri chiziq; b) to'g'ri chiziq va tekislik; c) ikki tekislik nechta umumiy nuqtaga ega bo'lishi mumkin?

4.66. a) Ikki to'g'ri chiziq; b) to'g'ri chiziq va tekislik; c) ikki tekislik; d) uchta tekislik yagona umumiy nuqtaga ega bo'lishi mumkinmi?

4.67. To'rtta nuqta bitta tekislikda yotmaydi. a) ulardan uchtasi bitta to'g'ri chiziqda yotishi mumkinmi? b) Ular orqali nechta tekislik o'tkazish mumkin?

4.68. *m* va *n* to‘g‘ri chiziqlar kesishadi, *d* to‘g‘ri chiziq esa *n* to‘g‘ri chiziqqa parallel. *m* va *d* to‘g‘ri chiziqlar o‘zaro qanday joylashishi mumkin?

4.69. *ABC* uchburchakning *C* uchidan o‘tuvchi va *AB* tomoniga parallel bo‘lgan nechta tekislik o‘tkazish mumkin?

4.70. *ABCD* va *ABKZ* parallelogrammlar turli tekisliklarda yotadi. Parallel to‘g‘ri chiziqlarni ko‘rsating:

A) *DA* va *KB*; B) *CD* va *KZ*; C) *BC* va *AZ*; D) *DA* va *ZA*; E) *CB* va *KB*.

4.71. *A* va *C* nuqtalar α tekislikka, *B* va *D* nuqtalar β tekislikka tegishli. *AC, CD, BD, AB, BC, AD* to‘g‘ri chiziqlardan qaysilari β tekislikni kesib o‘tadi?

4.72. *AB, AC, KB, KD* kesmalar α tekislikni kesib o‘tadi. *AK, AD, BD, KC, CD* to‘g‘ri chiziqlardan qaysilari α tekislikni kesib o‘tadi?

4.73. Bir tekislikda yotmagan *AB, AC* va *AD* to‘g‘ri chiziqlar α tekislikni B_1, C_1 va D_1 nuqtalarda kesib o‘tadi. B_1, C_1 va D_1 nuqtalar ketma-ket tutashtirilsa, qanday shakl hosil bo‘ladi?

4.74. α tekislikni kesib o‘tmaydigan *MN* kesma uchlaridan va o‘rtasidan parallel to‘g‘ri chiziqlar o‘tkazilgan. Agar bu to‘g‘ri chiziqlar α tekislikni, mos ravishda, M_1, N_1, va K_1 nuqtalarda kesib o‘tsa va KK_1= 9 sm, NN_1 = 15 sm bo‘lsa, MM_1 kesma uzunligini toping.

4.75. α tekislikning *P* va *Z* nuqtalaridan undan tashqarida uzunliklari *PK* = = 6 sm va *ZM* = 9 sm bo‘lgan parallel kesmalar tushirilgan. *MK* to‘g‘ri chiziq α tekislikni *O* nuqtada kesib o‘tadi. Agar *MK* = 6 sm bo‘lsa, *MO* kesma uzunligini toping.

4.76. Parallelogrammni parallel proyeksiyalashda kvadrat hosil bo‘lishi mumkinmi?

4.77. Uchburchakning parallel proyeksiyasi berilgan. Bu uchburchak medianalarining proyeksiyalari qanday yasaladi?

4.78. *MNZ* uchburchak va *MNPS* (*BC* – asos) parallelogramm bitta tekislikda yotmaydi. *Q* va *R* nuqtalar *CB* va *DA* kesmalarning o‘rtasi, *M* va *N* esa *DP* va *CZ* kesmalarning o‘rtasi. *MN* va *QR* to‘g‘ri chiziqlarning parallel ekanligini isbotlang.

4.79. $ABCDA_1B_1C_1D_1$ kubning (6- rasm) a) AA_1D_1D; b) BB_1C_1C; c) *ABCD*; d) DD_1C_1C; e) $B_1C_1D_1A_1$; f) ADD_1A_1 yoqlaridan qaysilari A_1B_1 to‘g‘ri chiziqqa parallel bo‘ladi?

4.80. *PRT* uchburchak berilgan. *PT* to‘g‘ri chiziqqa parallel α tekislik *PR* tomonni *S* nuqtada, *RT* tomonni *Q* nuqtada kesib o‘tadi (7- rasm). Agar *SR* = 7 sm, *SQ* = 3 sm va *SP*= 35 sm bo‘lsa, *PT* tomonni toping.

4.81. α tekislik *ABCD* trapetsiya asosi *AD* ga parllel hamda *AB* va *CD* tomonlarini *M* va *N* nuqtalarda kesib o‘tadi (8- rasm). *AD* = 20 sm, *MN* = 16 sm.

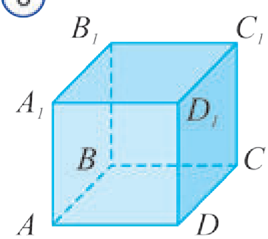
Agar M nuqta AB kesma o'rtasi va $AB = 8$ sm bo'lsa, trapetsiya perimetrini toping.

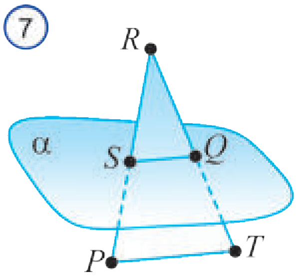
4.82. α tekislikning P va Z nuqtalariga undan tashqarida $PK = 6$ sm va $ZM = 9$ sm kesmalar o'tkazilgan. MK to'g'ri chiziq tekislikni O nuqtada kesib o'tadi. Agar $MK = 6$ sm bo'lsa, MO masofani toping.

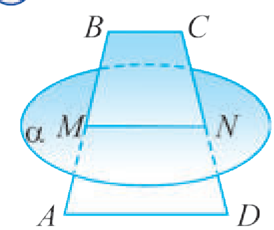
4.83. $ABCD$ to'g'ri to'rtburchkning AB tomoni α tekislikka parallel, AD tomoni esa bu tekislikka parallel emas. $ABCD$ va α tekisliklarning fazoda o'zaro joylashuvini aniqlang.

4.84. $ABCDA_1B_1C_1D_1$ to'g'ri burchakli parallelepipedning quyida berilgan yoqlaridan qaysilari A uchi va $ABCD$ yog'iga parallel bo'ladi:

A) D_1A_1AD; B) $D_1A_1B_1C_1$; C) ABB_1A_1; D) D_1C_1CD; E) D_1A_1BD?

6


7


8


4.85. Rombning ikki diagonali α tekislikka parallel. Pomb tekisligi va α tekislikning fazoda o'zaro joylashuvini aniqlang.

4.86. D nuqta ABC ucburchak tekisligida yotmaydi. K, Z va M nuqtalar, mos ravishda, DA, DB va DC kesmalarning o'rtalari. ABC va KZM tekisliklarning fazoda o'zaro joylashuvini aniqlang.

Tatbiqlar va amaliy kompetensiyalarni shakllantirish

1. Temiryo'l vagonlarining o'qlari bir-biriga nisbatan qanday joylashgan?
2. Temiryo'l vagonlarining o'qlari relslarga nisbatan qanday joylashgan?
3. Tevarak atrofdan parallel va ayqash to'g'ri chiziqlarga misollar keltiring.
4. Nima uchun yozuv stoli tortmalari ba'zida silliq ochilmaydi?
5. Nima uchun nasos porsheni uning ichida silliq harakatlanadi?
6. Tikuvchilki tasmasi yoki ixtiyoriy uzunlikdagi tayoq yordamida dahliz poli chekasiga qoqilgan reykalarning parallelligini qanday tekshirsa bo'ladi?
7. Yog'ochdan ishlangan brus (taxta) ning hamma yoqlari to'g'ri to'rtburchak shaklida. Uni ko'ndalang qirralari bo'ylab qanday arralamang, hosil bo'lgan hamma kesimlar parallelogramm bo'lishini isbotlang.

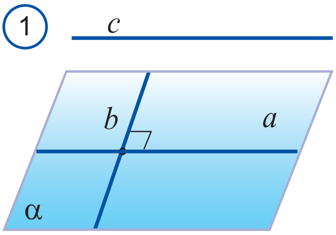
V BO‘LIM

FAZODA TO‘G‘RI CHIZIQLAR VA TEKISLIKLARNING PERPENDIKULARLIGI

15 FAZODA PERPENDIKULAR TO‘G‘RI CHIZIQ VA TEKISLIKLAR

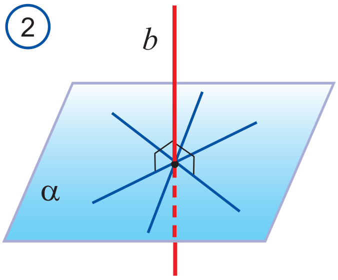
Eslatib o‘tamiz, fazoda berilgan ikki to‘g‘ri chiziq orasidagi burchak 90° ga teng bo‘lsa, ular o‘zaro *perpendikular to‘g‘ri chiziqlar* deyiladi. Perpendikular to‘g‘ri chiziqlar kesishuvchi va ayqash bo‘lishi mumkin. 1- rasmda a va b perpendikular to‘g‘ri chiziqlar kesishuvchi, b va c perpendikular to‘g‘ri chiziqlar esa ayqashdir. a va b to‘g‘ri chiziqlarning perpendikularligi $a \perp b$ tarzda yoziladi.

1



Tekislikdagi ixtiyoriy to‘g‘ri chiziqqa perpendikular to‘g‘ri chiziq *tekislikka perpendikular* deyiladi (2- rasm). α tekislik va b to‘g‘ri chiziqlarning perpendikularligi $b \perp \alpha$ tarzda yoziladi.

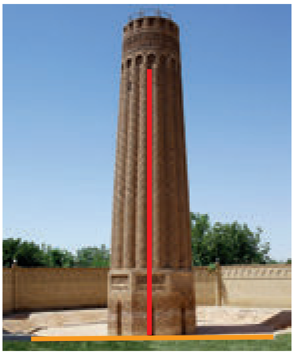
2



Tevarak atrofdan o‘zaro perpendikular shakllarga ko‘plab misollar keltirish mumkin. Odatda, uy devorlari va ustunlari, minoralar, chiroq ustunlari va simyog‘ochlar yerga nisbatan tik, ya’ni perpendikular qilib quriladi. Xonadagi shkaf, stol va sovitgichlar ham polga nisbatan tik qilib o‘rnatiladi (3- rasm).

3



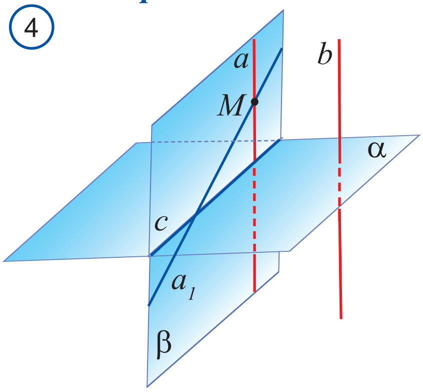


Endi fazodagi perpendikular to‘gri chiziqlarning ba’zi xossalari haqida to‘xtalamiz.

Agar a to‘g‘ri chiziq α tekislikda yotsa yoki unga parallel bo‘lsa, u holda α tekislikda yotgan va a to‘g‘ri chiziqqa parallell boshqa b to‘g‘ri chiziq ham topiladi. Shu bois, tekislikka perpendikular to‘g‘ri chiziq albatta bu tekislikni kesib o‘tadi. Teskari tasdiq ham o‘rinli bo‘ladi.

5.1- teorema. Agar ikki to‘g‘ri chiziq tekislikka perpendikular bo‘lsa, ular o‘zaro parallel bo‘ladi.

4



Isbot. a va b to‘g‘ri chiziqlar α tekislikka perpendikular bo‘lsin (4- rasm). Bu to‘g‘ri chiziqlarning o‘zaro parallel ekanligini isbotlaymiz.

a to‘g‘ri chiziqning biror M nuqtasidan b to‘g‘ri chiziqqa parallel a_1 to‘g‘ri chiziq o‘tkazamiz.

U holda, $a_1 \perp \alpha$ bo‘ladi.

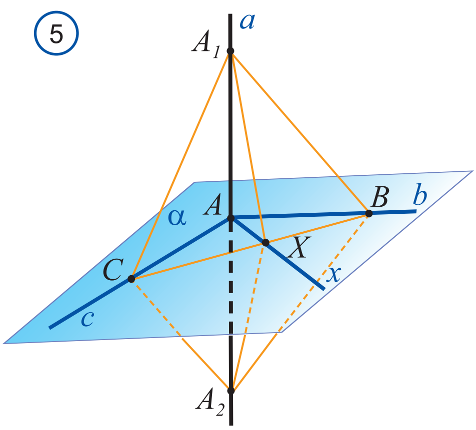
a va a_1 to‘g‘ri chiziqlarning ustma-ust tushishini ko‘rsatamiz. Aytaylik, unday bo‘lmasin, a va a_1 to‘g‘ri chiziqlar ustma-ust tushmasin. Unda a va a_1 to‘g‘ri chiziqlar yotgan β tekislikdagi M nuqtadan α va β tekisliklarning kesishish chizig‘i – c to‘g‘ri chiziqqa ikkita a va a_1 perpendikular to‘g‘ri chiziq o‘tadi. Buning esa bo‘lishi mumkin emas. Ziddiyat farazimizning noto‘g‘ri ekanligini ko‘rsatadi.

Demak, a va b to‘g‘ri chiziqlar o‘zaro parallel. ❑

Endi to‘g‘ri chiziqning tekislikka perpendikularlik alomatini keltiramiz.

5.2- teorema. Agar to‘g‘ri chiziq tekislikda yotgan ikki kesishuvchi to‘g‘ri chiziqqa perpendikular bo‘lsa, u tekislikka ham perpendikular bo‘ladi.

5



Isbot. a to‘g‘ri chiziq α tekislikda yotgan ikkita b va c to‘g‘ri chiziqqa perpendikular bo‘lsin. U holda a to‘g‘ri chiziq b va c to‘g‘ri chiziqlarning kesishish nuqtasi A orqali o‘tadi (5- rasm). a to‘g‘ri chiziqning α tekislikka perpendikular bo‘lishini isbotlaymiz.

α tekislikning A nuqtasi orqali ixtiyoriy x to‘g‘ri chiziq o‘tkazamiz va uning a to‘g‘ri chiziqqa perpendikular bo‘lishini ko‘rsatamiz. α tekislikda A nuqtadan o'tmaydigan, *b, c* va *x* to'g'ri chiziqlarni kesib o'tadigan ixtiyoriy to'g'ri chiziqni o'tkazamiz. Ularning kesishish nuqtalari, mos ravishda *B, C* va *X* nuqtalar bo'lsin.

a to'g'ri chiziqda *A* nuqtaning turli tomonlarida AA_1 va AA_2 kesmalarni qo'yamiz. Hosil bo'lgan A_1BA_2 va A_1CA_2 uchburchaklar teng yonli bo'ladi (buni mustaqil asoslang). Bundan A_1BC va A_2BC uchburchaklar teng bo'lishi kelib chiqadi (buni ham mustaqil asoslang). O'z navbatida, bundan A_1BX va A_2BX burchaklarning teng bo'lishi va nihoyat A_1BX va A_2BX uchburchaklarning ham teng bo'lishi kelib chiqadi (buni ham mustaqil asoslang).

Xususan, $A_1X = A_2X$ bo'ladi. Unda A_1XA_2 uchburchak teng yonli bo'ladi. Shuning uchun, uning *XA* medianasi uning balandligi ham bo'ladi. Bu esa, o'z navbatida, *x* to'g'ri chiziqning *a* to'g'ri chiziqqa perpendikular bo'lishini ko'rsatadi. Demak, *a* to'g'ri chiziq α tekislikka perpendikular. ❑

Bu teoremadan natija sifatida quyidagi xossalar kelib chiqadi. Ularni mustaqil asoslang.

5.3- teorema. ***Agar to'g'ri chiziq ikkita parallel tekislikning biriga perpendikular bo'lsa, ikkinchisiga ham perpendikular bo'ladi.***

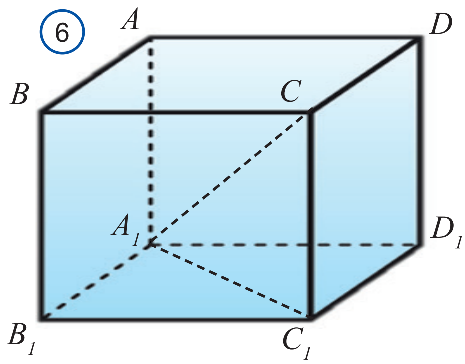
5.4- teorema. ***Agar ikkita tekislik bitta to'g'ri chiziqqa perpendikular bo'lsa, ular parallel bo'ladi.***

Quyida "mavjudlik va yagonalik teoremalari" deb ataluvchi xossalarni ham mustaqil isbotlash uchun keltiramiz.

5.5- teorema. ***Fazoning ixtiyoriy nuqtasidan berilgan to'g'ri chiziqqa perpendikular yagona tekislik o'tkazish mumkin.***

5.6- teorema. ***Fazoning ixtiyoriy nuqtasidan berilgan tekislikka perpendikular yagona to'g'ri chiziq o'tkazish mumkin.***

Natija (umumlashgan Pifagor teoremasi). ***To'g'ri burchakli parallelepiped diagonalining kvadrati uning uchta o'lchamlari kavadratlari yig'indisiga teng.***



$ABCDA_1B_1C_1D_1$ to'g'ri burchakli parallelepiped bo'lsin (6- rasm). CC_1 qirra $A_1B_1C_1D_1$ yoqqa perpendikular bo'lgani uchun A_1C_1C to'g'ri burchakli uchburchak bo'ladi. Unda Pifagor teoremasiga ko'ra,

$$A_1C^2 = CC_1^2 + A_1C_1^2 \qquad (1).$$

$A_1D_1C_1$ ham to'g'ri burchakli uchburchak. Yana Pifagor teoremasiga ko'ra,

$$A_1C_1^2 = A_1D_1^2 + D_1C_1^2 \qquad (2).$$

Unda, (1) va (2) ga ko‘ra: $A_1C^2 = CC_1^2 + A_1C_1^2 = CC_1^2 + A_1D_1^2 + D_1C_1^2$.

$A_1D_1 = B_1C_1$ bo‘lgani uchun $A_1C^2 = CC_1^2 + B_1C_1^2 + D_1C_1^2$. ❑

? *Mavzuga doir savollar va mashqlar*

1. *Fazoda qanday to‘g‘ri chiziqlar o‘zaro perpendikular bo‘ladi?*

2. *Ayqash to‘g‘ri chiziqlar perpendikular bo‘lishi mumkinmi?*
3. *7- rasmda qaysi shahar tasvirlangan? Unda siz qanday to‘g‘ri chiziqlarni va tekisliklarni ko‘ryapsiz? Rasmdan parallel, perpendikular va ayqash to‘g‘ri chiziqlarga misollar keltiring.*
4. *Qanday to‘g‘ri chiziq tekislikka perpendikular bo‘ladi?*
5. *Bitta tekislikka perpendikular to‘g‘ri chiziqlarning xossalarini ayting.*
6. *To‘g‘ri chiziq va tekisliklarning perpendikularlik alomatini ayting.*
7. *Parallel tekisliklarning biriga perpendikular bo‘lgan to‘g‘ri chiziqning xossasini ayting.*
8. *Bitta to‘g‘ri chiziqqa perpendikular bo‘lgan tekisliklarning xossasini ayting.*
9. *Umumlashgan Pifagor teoremasi nima haqida?*

5.1. *SB* kesma *ABCD* parallelogramm tekisligiga perpendikular (8- rasm). *SB* perpendikular bo‘lgan to‘g‘ri chiziqlarni ayting.

5.2. Qandaydir l to‘g‘ri chiziq *ABC* uchburchakning *AB* va *AC* tomonlariga perpendikular. l to‘g‘ri chiziq va *ABC* uchburchak tekisligining o‘zaro joylashuvini aniqlang.

a) l to‘g‘ri chiziq *ABC* tekislikni kesib o‘tadi, lekin unga perpendikular emas; b) l to‘g‘ri chiziq *ABC* tekislikka tegishli; c) l to‘g‘ri chiziq *ABC* tekislikka perpendikular; d) l to‘g‘ri chiziq *ABC* tekislikka parallel.

5.3. *KO* to‘g‘ri chiziq *ABCD* parallelogramm tekisligiga perpendikular (9- rasm). *KO* to‘g‘ri chiziqqa perpendikular to‘g‘ri chiziqni aniqlang

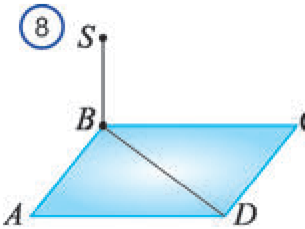
5.4. *MB* to‘g‘ri chiziq *ABC* uchburchakning *AB* va *BC* tomonlariga perpendikular (10- rasm). *X* nuqta *AC* tomonning ixtiyoriy nuqtas bo‘lsa, *MBX* uchburchak turini aniqlang.

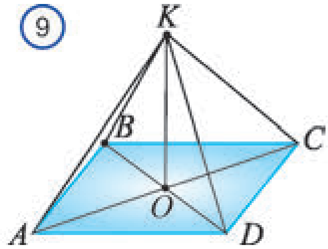
5.5. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepipedning AA_1C_1C va BB_1D_1D diagonal kesimlari o‘zaro perpendikular ekanligini isbotlang.

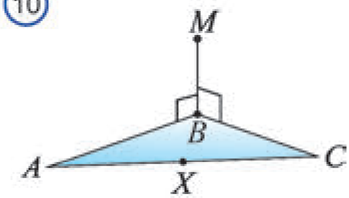
5.6. *ABCD* to‘rtburchakning tomonlari $A_1B_1C_1D_1$ to‘g‘ri to‘rtburchakning tomonlariga mos ravishda parallel. *ABCD* to‘g‘ri to‘rtburchak ekanligini isbotlang.

5.7. α tekislik *m* to‘g‘ri chiziqqa, *m* to‘g‘ri chiziq *n* to‘g‘ri chiziqqa parallel. Tekislikning *n* to‘g‘ri chiziqqa perpendikular bo‘lishini isbotlang.

5.8. *ABCD* trapetsiyaning *AB* asosi yotgan to‘g‘ri chiziq α tekislikka perpendikular. Bu trapetsiyaning *CD* asosi yotgan to‘g‘ri chiziq ham α tekislikka

8


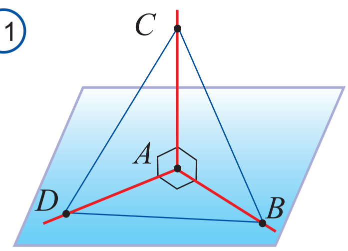
9


10


perpendikular bo‘lishini isbotlang.

5.9. Fazodagi to‘g‘ri chiziqning istalgan nuqtasidan unga perpendikular to‘g‘ri chiziq o‘tkazish mumkinligini isbotlang.

5.10. Fazodagi to‘g‘ri chiziqning istalgan nuqtasidan unga ikkita turli perpendikular to‘g‘ri chiziq o‘tkazish mumkinligini isbotlang.

11


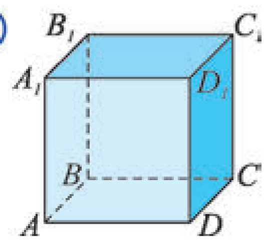
5.11. *AB, AC, AD* to‘g‘ri chiziqlar juft-jufti bilan o‘zaro perpendikular (11- rasm). Agar

1) $AB = 3$ sm, $BC = 7$ sm, $AD = 1{,}5$,sm;

2) $BD = 9$ sm, $BC = 16$ sm, $AD = 5$ sm;

3) $AB = b$ sm, $BC = a$ sm, $AD = d$ sm;

4) $BD = c$ sm, $BC = a$ sm, $AD = d$ sm bo‘lsa, *CD* kesma uzunligini toping.

12


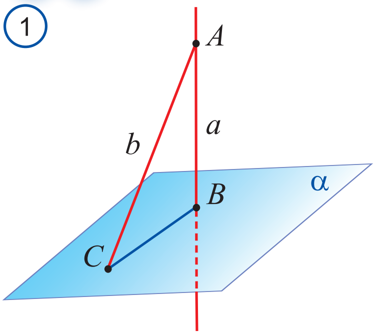
5.12. *ABCD* to‘g‘ri to‘rtburchakning *A* uchida uning tekisligiga perpendikular *AK* to‘g‘ri chiziq o‘tkazilgan. *K* nuqtadan to‘g‘ri to‘rtburchakning boshqa uchlarigacha masofa 6 m, 7 m, 9 m. *AK* masofani toping.

5.13. *A* va *B* nuqtalardan α tekislikka perpendikular va uni, mos ravishda, *C* va *D* nuqtalarda kesib o‘tuvchi to‘g‘ri chiziq o‘tkazilgan. Agar $AC = 3$ m, $BD = 2$ m va $CD = 2{,}4$ m bo‘lsa va *AB* kesma α tekislikni kesib o‘tmasa, *A* va *B* nuqtalar orasidagi masofani toping

5.14. 12- rasmda tasvirlangan kubning qirrasi: a) 4 sm; b) 8 sm bo‘lsa, AB_1C uchburchak perimetrini va DAC_1 uchburchak yuzini toping.

16 FAZODA PERPENDIKULAR, OG‘MA VA MASOFA

1



α tekislikka unda yotmagan A nuqtadan perpendikular a to‘g‘ri chiziq o‘tkazamiz (1-rasm). Bu to‘g‘ri chiziq tekislikni B nuqtada kesib o‘tsin. Shuningdek, tekislikning biror C nuqtasini A nuqta bilan tutashtiramiz. Natijada hosil bo‘lgan
AB kesma – *tekislikka tushirilgan perpendikular;*
AC kesma – *tekislikka tushirilgan og‘ma;*
BC kesma – *og‘maning tekislikdagi proyeksiyasi;*
B nuqta – *perpendikularning asosi;*
C nuqta – *og‘maning asosi* deb ataladi.

2

ABC uchburchak to‘g‘ri burchakli va unda AB katet, AC esa gipotenuza bo‘lgani uchun har doim $AB < AC$ bo‘ladi.

Demak, biror nuqtadan tekislikka tushirilgan perpendikularning uzunligi shu nuqtadan o‘tkazilgan ixtiyoriy og‘maning uzunligidan kichik bo‘ladi.

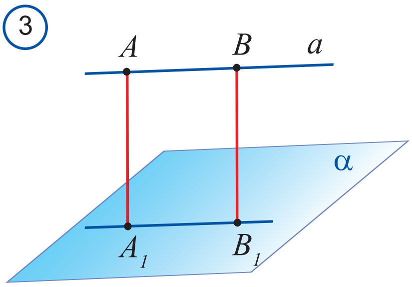
Nuqtadan tekislikkacha bo‘lgan masofa deb nuqtadan tekislikka tushirilgan perpendikular uzunligiga aytiladi.

Toshketdagi soat minorasining balandligi 30 m deyilganda, minoraning uchidan uning asos tekisligiga tushirilgan perpendikular uzunligi tushuniladi (2- rasm).

5.7- teorema. Agar to‘g‘ri chiziq tekislikka parallel bo‘lsa, u holda uning barcha nuqtalari tekislikdan baravar masofada bo‘ladi.

Isbot. a – berilgan to‘g‘ri chiziq va α – berilgan tekislik bo‘lsin (3-rasm). a to‘g‘ri chiziqda ikkita A va B nuqtani olamiz. Ulardan α tekislikka perpendikularlar tushuramiz. Bu perpendikularlar asosi, mos ravishda, A va B nuqtalar bo‘lsin. Unda A va B nuqtalardan α tekislikkacha bo‘lgan masofalar, mos ravishda, AA_1 va BB_1 kesmalar bo‘ladi. 4.6-teoremaga ko‘ra, AA_1 va BB_1 kesmalar parallel bo‘ladi.

3

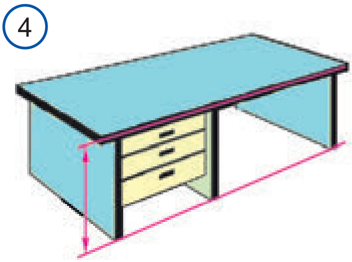


Demak, ular bitta tekislikda yotadi. Bu tekislik α tekislikni A_1B_1 to‘g‘ri chiziq bo‘ylab kesadi. a to‘g‘ri chiziq A_1B_1 to‘g‘ri chiziqqa parallel bo‘ladi, chunki u α tekislikni kesib o‘tmaydi.

Shunday qilib, ABA_1B_1 to‘rtburchakning qarma-qarshi tomonlari parallel. Demak, u parallelogramm. Bu parallelogrammda $AA_1 = BB_1$. ❑

To‘g‘ri chiziqdan unga parallel bo‘lgan tekislikkacha bo‘lgan masofa deb to‘g‘ri chiziqning ixtiyoriy nuqtasidan shu tekislikkacha bo‘lgan masofaga aytiladi.

4



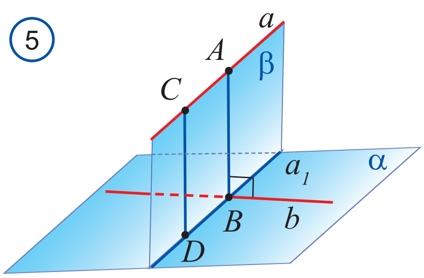
Tekislikning ixtiyoriy ikki nuqtasidan unga parallel bo‘lgan tekislikkacha bo‘lgan masofa bir xil bo‘ladi. Bu xossa oldingi teorema isbotiga o‘xshash isbotlanadi.

Ikki parallel tekisliklar orasidagi masofa deb bir tekislikning ixtiyoriy nuqtasidan ikkinchi tekislikkacha bo‘lgan masofaga aytiladi.4- rasmda tasvirlangan stolning balandligi pol va stol tekisliklari orasidagi masofaga teng bo‘ladi.

5.8- teorema. Ikki ayqash to‘g‘ri chiziq yagona umumiy perpendikularga ega bo‘ladi.

Isbot. a va b ayqash to‘g‘ri chiziqlar bo‘lsin (5- rasm). Bu to‘g‘ri chiziqlarda shunday A va B nuqtalarni talash mumkinligini ko‘rsatamizki, AB to‘g‘ri chiziq ham a ga, ham b ga perpendikular bo‘ladi. α tekislik b to‘g‘ri chiziqdan o‘tuvchi va a to‘g‘ri chiziqqa parallel bo‘lsin. a to‘g‘ri chiziqda C nuqtani olamiz va undan α tekislikka CD perpendikular tushuramiz. Kesishuvchi a va CD to‘g‘ri chiziqlardan β tekislikni o‘tkazamiz. a_1 to‘g‘ri chiziq α va β tekisliklarning kesishish chizig‘i bo‘lsin.

5



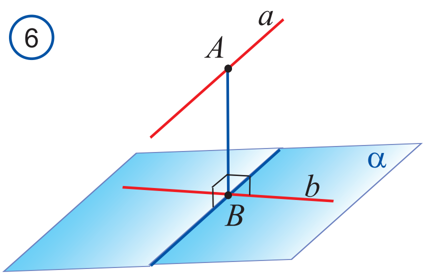
$a_1 \parallel a$ bo‘lgani uchun a_1 va b to‘g‘ri chiziqlar qandaydir B nuqtada kesishsadi. B nuqtadan β tekislikda yotuvchi, a to‘g‘ri chiziqqa BA perpendikular tushuramiz.

Natijada, AB va CD to‘g‘ri chiziqlarning har ikkalasi ham β tekislikda yotadi va a to‘g‘ri chiziqqa perpendikular bo‘ladi. Shuning uchun, $AB \parallel CD$ va $AB \perp \alpha$ bo‘ladi.

Demak, $AB \perp a$ va $AB \perp b$ bo‘ladi. AB izlayotgan to‘g‘ri chiziq bo‘lib, u a va b ayqash to‘g‘ri chiziqlarning har ikkalasiga ham perpendikular bo‘ladi.

Umumiy perpendikularning yagonaligini mustaqil isbotlang. ❑

Ikki ayqash to‘g‘ri chiziq orasidagi masofa deb ularning umumiy perpendikulari uzunligiga aytiladi.



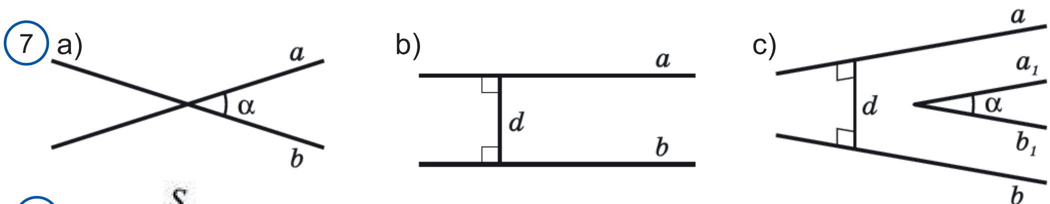
Yuqoridagi teoremadan quyidagi xulosa kelib chiqadi:

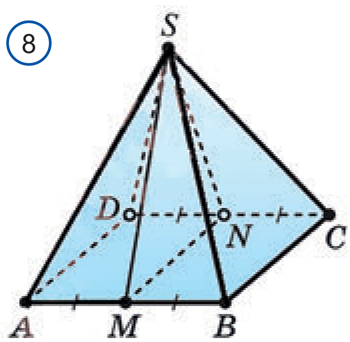
Ikki ayqash *a* va *b* to‘g‘ri chiziq orasidagi masofa (6- rasm) *a* to‘g‘ri chiziqning istalgan nuqtasidan *b* to‘g‘ri chiziq yotgan va *a* to‘g‘ri chiziqqa parallel bo‘lgan α tekislikkacha bo‘lgan masofaga teng bo‘ladi.

Yuqoridagilarga asoslanib, endi biz fazoda ikki to‘g‘ri chiziqning o‘zaro joylashishini sonlar yordamida tavsiflashimiz mumkin.

Agar fazoda ikki to‘g‘ri chiziq:

- o‘zaro kesishsa, ular orasidagi α burchak (7.a- rasm),
- o‘zaro parallel bo‘lsa, ular orasidagi *d* masofa (7.b- rasm),
- o‘zaro ayqash bo‘lsa, ular orasidagi α burchak va orasidagi *d* masofa (7.c- rasm) mazkur to‘g‘ri chiziqlarning o‘zaro joylashishini sonli tavsiflaydi.





Masala. To‘rtburchakli *SABCD* piramidaning barcha qirralari *a* ga teng. Uning *AB* va *SC* qirralari orasidagi masofani toping (8- rasm).

Yechish. 4.8- teoremaga ko‘ra, *AB* va *SC* qirralarida shunday *X* va *Y* nuqtalar borki, *XY* to‘g‘ri chiziq *AB* va *SC* qirralarning har ikkalasiga ham perpendikular bo‘ladi. Shuningdek, *XY* to‘g‘ri chiziq *SC* to‘g‘ri chiziq yotgan va *AB* to‘g‘ri chiziqqa parallel bo‘lgan tekislikka ham perpendikular bo‘ladi.

Aytaylik, α tekislik *S* nuqtadan o‘tuvchi va *AB* to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik bo‘lsin. Bu tekislik *AB* va *CD* qirralarning o‘rtalari – *M* va *N* nuqtalardan o‘tadi. Unda $XY \parallel \alpha$ va *XY* kesmaning α tekislikdagi proyeksiyasi *XY* kesmaga teng bo‘ladi.

Endi *X* va *Y* nuqtalarning α tekislikdagi qaysi nuqtalarga proyeksiyalanishini aniqlaymiz.

$AB \perp \alpha$ bo‘lgani uchun *AB* qirraning barcha nuqtalari *M* nuqtaga proyeksiyalanadi. Demak, *X* nuqta *M* nuqtaga proyeksiyalanadi.

S va *C* nuqtalar, mos ravishda, *S* va *N* nuqtalarga proyeksiyalangani uchun, *SC* kesma *SN* kesmaga o‘tadi. *SN* to‘g‘ri chiziq *AB* to‘g‘ri chiziqqa parallel tekislikda yotgani uchun, izlanayotgan, *XY* kesmaning proyeksiyasi – *SN* to‘g‘ri chiziqqa *M* nuqtadan tushurilgan perpendikulardan iborat bo‘ladi.

Bu perpendikular uzunligi *d* ni asosi *a* va yon tomoni $\frac{a\sqrt{3}}{2}$ ga teng bo‘lgan *SMN* teng yonli uchburchak yuzi ifodalaridan foydalanib topamiz.

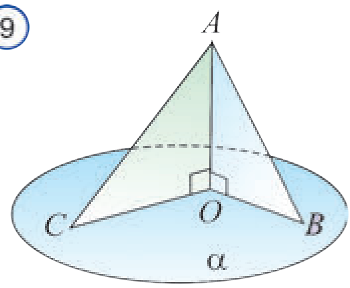
Bir tomondan bu uchburchak yuzi: $\frac{a}{2}\cdot\frac{a\sqrt{2}}{2}$ ga teng, ikkinchi tomondan esa $\frac{1}{2}\cdot\frac{a\sqrt{3}}{2}d$ ga teng. Bu ikkita tenglikdan $d = \frac{a\sqrt{6}}{3}$ bo‘ladi.

Mavzuga doir savollar va mashqlar

1. *Tekislikka tushurilgan perpendikular va og‘maga ta’rif bering*
2. *Og‘maning tekislikdagi proyeksiyasi deb nimaga aytiladi?*
3. *Nuqtadan tekislikkacha bo‘lgan masofa qanday aniqlanadi?*
4. *Tekislikka parallel bo‘lgan to‘g‘ri chiziq va tekislik orasidagi masofa qanday topiladi?*
5. *Ikki parallel tekisliklar orasidagi masofa qanday aniqlanadi?*
6. *Ikki ayqash to‘g‘ri chiziqlar orasidagi masofa qanday aniqlanadi?*
7. *Fazoda ikki to‘g‘ri chiziqning o‘zaro joylashishini qaysi sonli kattaliklar aniqlaydi?*

5.15. *A, B, Q* nuqtalar α tekislikka tegishli, *M* nuqta esa unga tegishli emas va $MQ \perp \alpha$. *MA, AQ, MQ, BQ, MB* kesmalarning qaysi biri a) perpendikular; b) og‘ma; c) og‘ma proyeksiyasi ekanligini aniqlang.

9



5.16. *A* nuqtadan *a* tekislikka *AB* va *AC* og‘malar va *AO* perpendikular o‘tkazilgan (9- rasm). Agar *AB* = 2,5 sm, *AC* = 3 sm bo‘lsa, og‘malarning proyeksiyalarini o‘zaro taqqoslang.

5.17. Nuqtadan tekislikka ikkita og‘ma tushirilgan (9- rasm). Agar og‘malarning biri ikkinchisidan 26 sm uzun, proyeksiyalari esa 12 sm va 40 sm bo‘lsa, bu og‘malarning uzunliklarini toping.

5.18. Uchburchakka tashqi chizilgan aylana markazidan uchburchak tekisligiga perpendikular to‘g‘ri chiziq o‘tkazilgan. Bu to‘g‘ri chiziqning har bir nuqtasi uchburchak uchlaridan baravar uzoqlikda yotishini isbotlang.

5.19. Yuzi a) 21 sm^2; b) 96 sm^2; c) 44 sm^2; d) 69 sm^2; e) 156 sm^2 bo‘lgan *ABCD* kvadrat tekisligiga uzunligi 10 sm bo‘lgan *DM* perpendikular tushurilgan. *MA* og‘maning uzunligini toping.

5.20. To‘g‘ri burchagi C bo‘lgan ABC uchburchakning o‘tkir burchagi uchidan uchburchak tekisligiga perpendikular AD to‘g‘ri chiziq o‘tkazilgan. Agar $AC = c$, $BC = b$ va $AD = c$ bo‘lsa, D nuqtadan B va C uchlargacha bo‘lgan masofalarni toping.

5.21. Bir-biridan 3,4 m uzoqlikda bo‘lgan vertikal ustunlarning yuqori uchlari to‘sin bilan tutashtirilgan. Ustunlarning balandliklari 5,8 m va 3,9 m bo‘lsa, to‘sin uzunligini toping.

5.22. 15 m uzunlikdagi telefon simi simyog‘ochga yer sathidan 8 m balandlikda mahkamlangan va undan balandligi 20 m bo‘lgan ko‘pqavatli uy tomiga tarang tortilgan. Uy bilan ustun orasidagi masofani toping.

5.23. Tekislikka P nuqtadan tushirilgan PQ perpendikular uzunligi 1 ga, PA va PB og‘malar uzunliklari esa 2 ga teng. C nuqta AB kesma o‘rtasi. Agar a) $\angle APB = 90°$; b) $\angle APB = \beta$ bo‘lsa, QC kesma uzunligini toping.

5.24. $ABCD$ parallelogrammning o‘tmas B burchagi uchidan uning tekisligiga perpendikular bo‘lgan BH kesma tiklangan. Agar $AH = 5$ sm, $HD = HC = 8{,}5$ sm, $AC = 1{,}5\sqrt{33}$ bo‘lsa, parallelogramm tomonlarini toping.

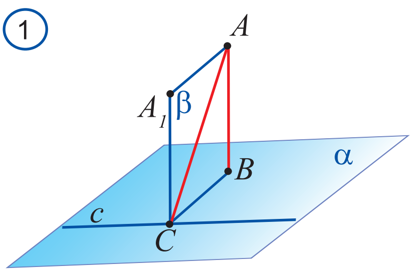
5.25. M nuqta tomoni 60 sm bo‘lgan muntazam ABC uchburchakning har bir uchdan 40 sm masofada joylashgan. ABC uchburchak tekisligidan M nuqtagacha bo‘lgan masofani toping.

17 UCH PERPENDIKULARLAR HAQIDAGI TEOREMA

5.9- teorema. ***Agar tekislikka tushirilgan og‘maning asosidan o‘tuvchi to‘gri chiziq og‘maning proyeksiyasiga perpendikular bo‘lsa, u holda u og‘maning o‘ziga ham perpendikular bo‘ladi.***

Isbot. Aytaylik, AB kesma α tekislikka tushurilgan perpendikular, AC kesma esa og‘ma bo‘lsin. c to‘g‘ri chiziq α tekislikda yotuvchi, C nuqtadan o‘tuvchi va og‘ma proyeksiyasiga perpendikular bo‘lgan to‘g‘ri chiziq bo‘lsin (1- rasm). AB ga parallel A_1C to‘gri chiziqni o‘tkazamiz. Bu to‘g‘ri chiziq α tekislikka perpendikular bo‘ladi.

1



AB va AC_1 to‘g‘ri chiziqlar orqali β tekislikni o‘tkazamiz. c to‘g‘ri chiziq CA_1 to‘g‘ri chiziqqa perpendikular bo‘ladi. U shartga ko‘ra, CB to‘g‘ri chiziqqa ham perpendikular edi. Unda c to‘g‘ri chiziq β tekislikka ham perpendikular bo‘ladi.

Demak, c to‘g‘ri chiziq β tekislikda yotgan AC og‘maga ham perpendikular bo‘ladi. ❑

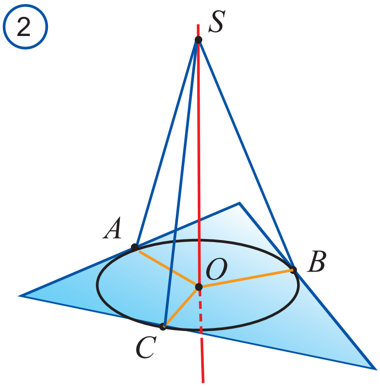
Mazkur teoremada uchta perpendikularlar haqida gap borayotgani uchun u “Uch perpendikularlar haqidagi teorema” nomini olgan. Bu teoremaga teskari bo‘lgan teorema ham o‘rinli bo‘ladi. Uni mustaqil isbotlang.

5.10- teorema. Agar tekislikka tushirilgan og‘maning asosidan o‘tuvchi to‘g‘ri chiziq og‘maga perpendikular bo‘lsa, u holda u og‘maning proyeksiyasiga ham perpendikular bo‘ladi.

1- masala. Uchburchakka ichki chizilgan aylana markazidan uchburchak tekisligiga perpendikular to‘g‘ri chiziq o‘tkazilgan (2- rasm). Bu to‘g‘ri chiziqning ixtiyoriy nuqtasi uchburchak tomonlaridan baravar uzoqlikda yotishini isbotlang.

Isbot. Aytaylik, A, B, C – uchburchak tomonlarining aylana bilan kesishish nuqtalari, O – aylana markazi, S esa perpendikulardagi ixtiyoriy nuqta bo‘lsin.

2



OA uchburchak tomoniga perpendikular bo‘lgani uchun, uch perpendikularlar haqidagi teoremaga ko‘ra, OA ham bu tomonga perpendikular bo‘ladi. Unda SAO to‘g‘ri burchakli uchburchak bo‘ladi. Bu uchburchakda Pifagor teoremasiga ko‘ra,

$SA = \sqrt{AO^2 + OS^2} = \sqrt{r^2 + OS^2}$,

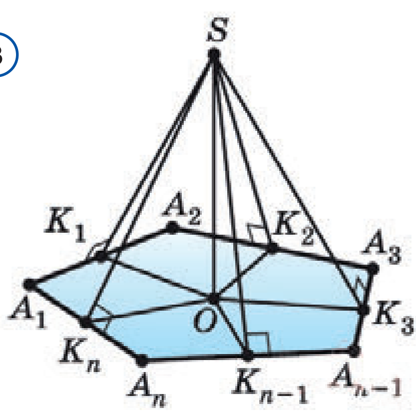
bu yerda r – aylana radiusi.

Xuddi shunga o‘xshash, SBO to‘g‘ri burchakli uchburchakdan $SB = \sqrt{r^2 + OS^2}$ va SCO to‘g‘ri burchakli uchburchakdan esa $SC = \sqrt{r^2 + OS^2}$ ekanligini topamiz.

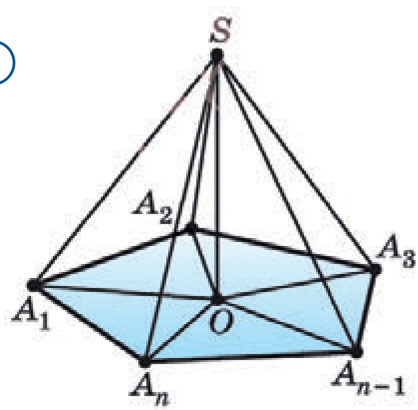
Demak, $SA = SB = SC$. ❑

Yuqorida keltirilgan 3– 4- rasmlar asosida 1- masalaga o‘xshash va ixtiyoriy ko‘pburchak uchun umumiyroq hollarni mustaqil isbotlash uchun keltiramiz.

3



4

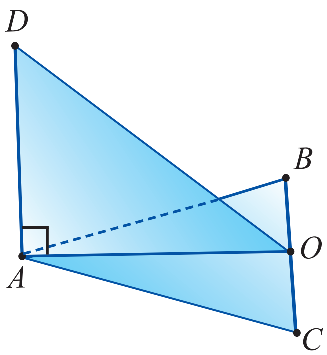


2- masala. Fazodagi nuqta ko'pburchakning tomonlaridan baravar uzoqlikda joylashgan bo'lib, undan ko'pburchak tekisligiga perpendikular tushurilgan. Bu perpendikular asosi ko'pburchakka ichki chizilgan aylana markazi bilan ustma-ust tushishini isbotlang (3- rasm).

3- masala. Fazodagi nuqta ko'pburchakning uchlaridan baravar uzoqlikda joylashgan bo'lib, undan ko'pburchak tekisligiga perpendikular tushurilgan. Bu perpendikular asosi ko'pburchakka tashqi chizilgan aylana markazi bilan ustma-ust tushishini isbotlang (4- rasm).

4- masala. ABC uchburchak tekisligiga uning A nuqtasidan perpendikular tushirilgan (5- rasm). Agar $AB = 13$, $BC = 20$, $AC = 11$ va $AD = 36$ ga teng bo'lsa, D nuqtadan BC to'g'ri chiziqqacha bo'lgan masofani toping.

5



Yechish. Izlanayotgan masofa D nuqtadan BC tomonga tushurilgan perpendikular uzunligiga teng bo'ladi. Bu kesmani tushurish uchun uning BC tomondagi asosini topish lozim bo'ladi. Buning uchun ABC uchburchakning A uchidan BC tomoniga AO balandlikni tushuramiz: $AO \perp BC$.

Unda uch perpendikularlar haqidagi teoremaga ko'ra, $BC \perp DO$ bo'ladi. Demak, DO izlanayotgan kesma ekan.

Endi DO kesmaning uzunligini topamiz. Buning uchun, oldin ABC uchburchak yuzini Geron formulasidan foydalanib topamiz:

$p = (a + b + c) : 2 = (20 + 11 + 13) : 2 = 22;$

$S = \sqrt{p \cdot (p-a) \cdot (p-b) \cdot (p-c)} = \sqrt{22 \cdot (22-20) \cdot (22-11) \cdot (22-13)} = 66.$

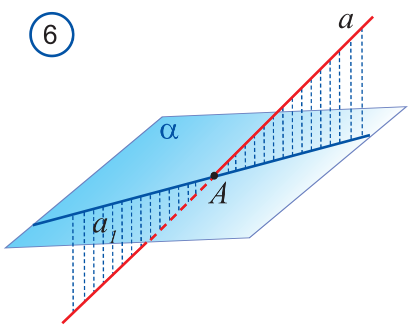
$DO = 2\,S / a = (2 \cdot 66) : 20 = 6{,}6.$

ADO to'g'ri burchakli uchburchakda, Pifagor teoremasiga ko'ra

$DO = \sqrt{AD^2 + AO^2} = \sqrt{36^2 + 6{,}6^2} = 36{,}6.$

Aytaylik, α tekislik va uni kesib o'tuvchi va bu tekislikka perpendikular bo'lmagan a to'g'ri chiziq berilgan bo'lsin (6- rasm). a to'g'ri chiziqning har bir nuqtasidan perpendikularlar tushuramiz. Bu perpendikularlarning asoslari a_1 to'g'ri chiziqni tashkil qiladi.

6

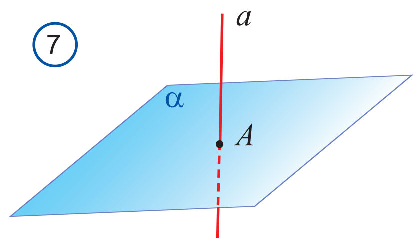


a_1 to'g'ri chiziq a to'g'ri chiziqning α tekislikdagi *proyeksiyasi* deb ataladi.

a to'g'ri chiziq va α tekislik orasidagi burchak deb, to'g'ri chiziq bilan uning bu tekislikdagi proyeksiyasi orasidagi burchakka aytiladi.

Agar to‘g‘ri chiziq tekislikka perpendikular bo‘lsa (7- rasm), u bilan tekislik orasidagi burchak 90° ga, agar parallel bo‘lsa, bu to‘g‘ri chiziq bilan tekislik orasidagi burchak 0° ga teng deb olinadi.

7



Mavzuga doir savollar va mashqlar

1. *Uch perpendikularlar haqidagi teoremani sharhlang. Nima sababdan u shunday nomlangan?*
2. *Uch perpendikularlar haqidagi teoremaga teskari teoremani ayting va izohlang.*
3. *To‘g‘ri chiziq va tekislik orasidagi burchak qanday aniqlanadi?*
4. *Tekislik va unga perpendikular to‘g‘ri chiziq orasidagi burchak necha gradus?*

5.26. *A* nuqta tomoni *a* ga teng bo‘lgan teng tomonli uchburchakning uchlaridan *a* masofada yotadi. *A* nuqtadan uchburchak tekisligigacha bo‘lgan masofani toping.

5.27. α tekislikning tashqarisidagi *S* nuqtadan unga uchta teng *SA, SB, SC* og‘ma va *SO* perpendikular o‘tkazilgan. Perpendikularning *O* asosi *ABC* uchburchakka tashqi chizilgan aylananing markazi bo‘lishini isbotlang.

5.28. Teng tomonli uchburchakning tomonlari 3 m ga teng. Uchburchak har bir uchidan 2 m masofada bo‘lgan nuqtadan uchburchak tekisligigacha bo‘lgan masofani toping.

5.29. Teng yonli uchburchakda asosi va balandligi 4 m ga teng. Berilgan nuqta uchburchak takisligidan 6 m masofada va uning uchlaridan bit xil masofada yotadi. Shu masofani toping.

5.30. *A* nuqtadan kvadratning uchlarigacha bo‘lgan masofa *a* ga teng. Kvardatning tomoni *b* ga teng bo‘lsa, *A* nuqtadan kvadrat tekisligigacha bo‘lgan masofani toping.

5.31. Berilgan nuqtadan tekislikka o‘tkazilgan berilgan uzunlikdagi og‘malar asoslarining geometrik o‘rnini toping.

5.32. Berilgan nuqtadan tekislikka uzunliklari 10 sm va 17 sm bo‘lgan ikkita og‘ma o‘tkazilgan. Bu og‘malar proyeksiyasining ayirmasi 9 sm ga teng. Og‘malar proyeksiyalarini toping.

5.33. Nuqtadan tekislikka ikkita og‘ma o‘tkazilgan. Agar: 1) ulardan biri ikkinchisidan 26 sm uzun, og‘malarning proyeksiyaslari 12 sm va 40 sm bo‘lsa; 2) og‘malar uzunliklari 1 : 2 nisbatda bo‘lib, ularning proyeksiyalari 1 sm va 7 sm ga teng bo‘lsa, og‘malarning uzunliklarini toping.

5.34. α tekislikdan *d* masofada yotgan *A* nuqtadan tekisik bilan 30° burchak tashkil qiluvchi *AB* va *AC* og‘malar o‘tkazilgan.Ularning α tekislikka proyeksiyalari o‘zaro 120° li burchak tashkil qiladi. *BC* kesma uzunligini toping.

5.35. Agar to‘g‘ri burchakli uchburchakning katetlaridan biri tekislikka tegishli, ikkikchisi esa u bilan 45° li burchak tashkil qilsa, gipitenuza bu tekislik bilan 30° li burchak tashkil qilishini isbotlang.

5.36. a og‘ma α tekislik bilan 45° li burchak tashkil qiladi, tekislikning b to‘g‘ri chizig‘i esa og‘ma proyeksiyasi bilan 45° li burchak tashkil qiladi. a va b to‘g‘ri chizqlar orasidagi burchakning 30° ga teng ekanligini isbotlang.

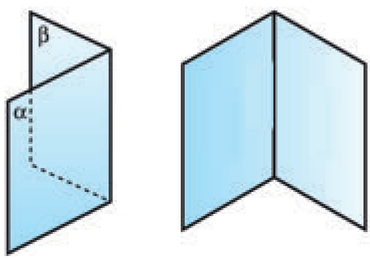
5.37. P nuqta tomoni a ga teng $ABCD$ kvadratning har bir uchidan a masofada yotadi. Kvadrat tekisligi va AP to‘g‘ri chiziq orasidagi burchakni toping.

5.38. Uchburchakli piramidaning hamma qirralari o‘zaro teng. Piramidaning qirrasi va bu qirra tegishli bo‘lmagan yog‘i orasidagi burchakni toping.

5.39. To‘g‘ri burchakli parallelepipedning o‘lchamlari a, b va c ga teng. Parallelepiped diagonali bilan uning yoqlari diagonallari orasidagi masofani toping.

18 FAZODA TEKISLIKLARNING PERPENDIKULARLIGI

1



Ikkita yarimtekislik va ularni chegaralab turgan umumiy to‘g‘ri chiziqdan iborat geometrik shakl *ikki yoqli burchak* deb ataladi (1- rasm). Yarimtekisliklar ikki yoqli burchakning *yoqlari*, ularni chegaralovchi to‘g‘ri chiziq esa ikki yoqli burchakning *qirrasi* deb ataladi.

Ikki yoqli burchaklar haqida tevarak atrofdagi quyidagi narsalar tasavvur beradi (2- rasm): kitob, noutbuk , ochilq eshik va imorat tomi.

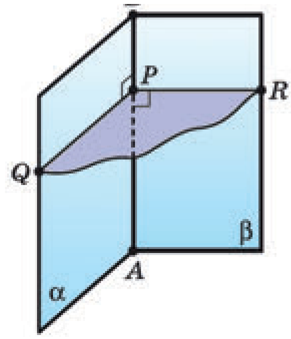
Ikki yoqli burchak qirrasining ixtiyoriy nuqtasidan uning yoqlarida yotuvchi va bu qirraga perpendikular bo‘lgan nurlarni chiqaramiz. Bu nurlar hosil qilgan burchak ikki yoqli burchakning *chiziqli burchagi* deb ataladi (3- rasm).

Ta’rifdan ko‘rinadiki, ikki yoqli burchakning chiziqli burchagi qirrada tanlangan nuqta bilan aniqlanadi va cheksiz ko‘p bo‘ladi. Shunday bo‘lsada, ikki yoqli burchakning chiziqli burchagi kattaligi qirrada tanlangan nuqtaga bog‘liq emas, ya’ni ularning hammasi o‘zaro teng bo‘ladi.

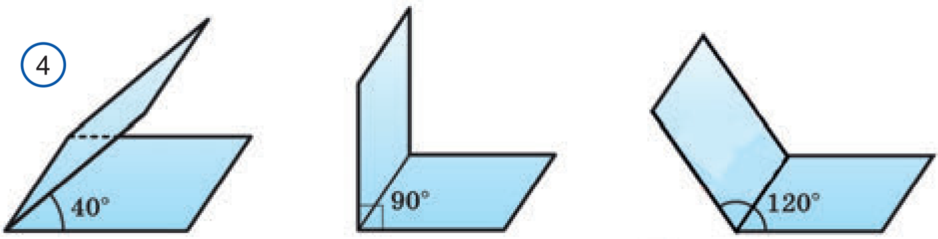
Ikki yoqli burchaklar kattaligi uning chiziqli burchagi kattaligi bilan aniqlanadi. Chiziqli burchaklarning o‘tkir, to‘g‘ri, o‘tmas va yoyiq bo‘lishiga qarab, ikki yoqli burchaklar ham, mos ravishda, o‘tkir, to‘g‘ri, o‘tmas va yoyiq ikki yoqli burchaklarga ajratiladi. 4- rasmda turli xil ikki yoqli burchaklar tasvirlangan.

2

3



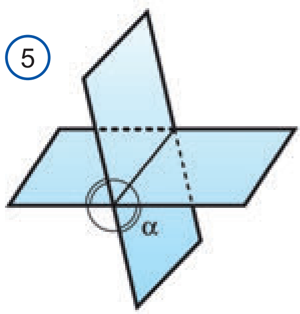
4



Ikki kesishuvchi tekislik butun fazoni umumiy qirraga ega bo‘lgan to‘rtta ikki yoqli burchakka ajratadi (5- rasm). Bu ikki yoqli burchaklarning biri α ga teng bo‘lsa, ulardan yana bittasining qiymati ham α ga teng bo‘ladi. Qolgan ikkitasining qiymati esa 180° – α ga teng bo‘ladi.

Mazkur ikki yoqli burchaklar ichida 90° dan kichigi ham bo‘ladi. Shu burchakning qiymati kesishuvchi *tekisliklar orasidagi burchak* deb olinadi.

5



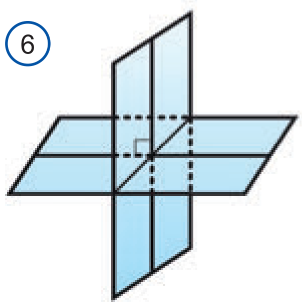
Agar ikki yoqli burchaklarning biri to‘g‘ri, ya’ni 90° ga teng bo‘lsa, qolgan uchtasi ham to‘g‘ri bo‘ladi (6- rasm).

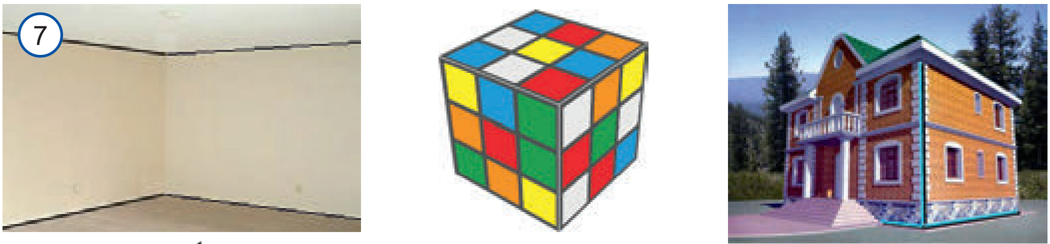
To‘g‘ri burchak ostida kesishuvchi tekisliklar – *perpendikular tekisliklar* deb ataladi.

Perpendikular tekisliklarga tevarak atrofdan misol sifatida xona poli va devorlarini, umumiy qirraga ega xona devorlari, umumiy qirraga ega Rubik kubi yoqlari, yer sathi va uy devorlari hamda uyning bir-biriga tutashgan devorlarini misol tariqasida keltirish mumkin (7- rasm).

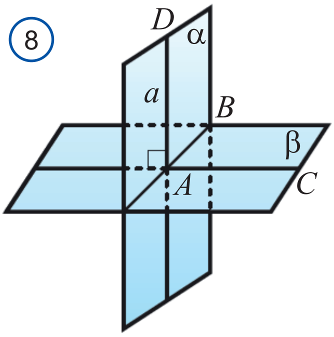
α va β tekisliklarning perpendikularligi to‘g‘ri chiziqlardagi kabi “⊥” belgi yordamida, α ⊥ β tarzda yoziladi.

6





8



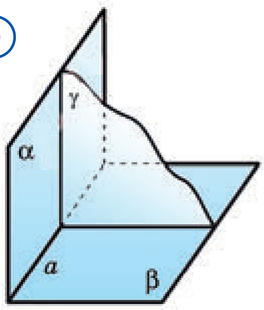
Endi perpendikular tekisliklarning xossalari haqida to‘xtalamiz. Quyidagi teorema – “tekisliklarning perpendikularlik alomati” deb nomlanadi.

5.11- teorema. Agar tekisliklardan biri ikkinchisiga perpendikular bo‘lgan to‘g‘ri chiziqdan o‘tsa, bunday tekisliklar o‘zaro perpendikular bo‘ladi.

Isbot. Aytaylik, α va β tekisliklar berilgan bo‘lib, α tekislik β tekislikka perpendikular bo‘lgan *a* to‘g‘ri chiziqdan o‘tsin (8- rasm). β tekislik bilan *a* to‘g‘ri chiziqning kesishish nuqtasi *A* bo‘lsin. *a*⊥*b* ekanligini isbotlaymiz.

α va β tekisliklar *AB* to‘g‘ri chiziq bo‘ylab kesishishyapti. Unda *AB*⊥*a* bo‘ladi, chunki shartga ko‘ra *b*⊥*a*. β tekislikda yotgan va *AB* to‘g‘ri chiziqqa perpendikular bo‘lgan *AC* to‘g‘ri chiziqni otkazamiz. Natijada, hosil bo‘lgan *DAC* burchak αβ ikki yoqli burchakning chiziqli burchagi bo‘ladi. Shartga ko‘ra, *a*⊥β. Unda *DAC* to‘g‘ri burchak. Demak, α⊥β. ❑

9



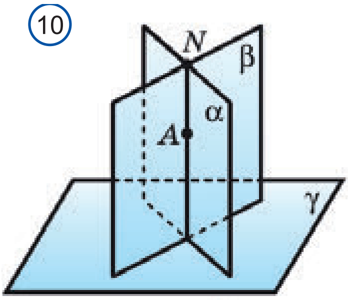
Bu teoremadan quyidagi natija kelib chiqadi.

Natija. Agar tekisliklar ikki tekislikning kesishish chizig‘iga perpendikular bo‘lsa, bu tekisliklarning har biriga ham perpendikular bo‘ladi (9- rasm).

5.11- teoremaga teskari teorema ham o‘rinli bo‘ladi. Uni isbotsiz keltiramiz.

5.12- teorema. Agar ikki perpendikular tekisliklardan birining biror nuqtasidan ikkinchisiga perpendikular to‘g‘ri chiziq o‘tkazilsa, bu to‘g‘ri chiziq birinchi tekislikda yotadi.

10

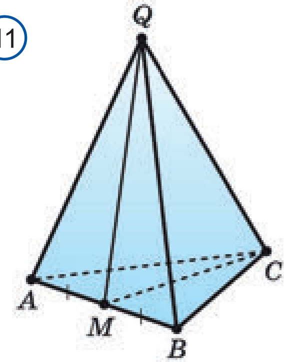


Natija. Agar ikki perpendikular tekislik uchinchi tekislikka perpendikular bo‘lsa, ularning kesishish chizig‘i ham bu tekislikka perpendikular bo‘ladi (10- rasm).

1- masala. *M* nuqta *QABC* muntazam piramida asosidagi qirrasining o‘rtasi bo‘lsa (11- rasm), *QCM* tekislik piramida asosi tekisligi ABC ga perpendikular ekanligini isbotlang.

Isbot. AB kesma teng yonli AQB va ACB uchburchaklarning asosi bo‘lgani uchun bu uchburchaklar medianalari QM va CM ga ham perpendikular bo‘ladi. Shu bilan birga, AB kesma QCM tekislikka ham perpendikular bo‘ladi. Unda 5.12- teoremaga ko‘ra, ABC tekislik QCM tekislikka perpendikular bo‘ladi. ❑

11



2- masala. $QABC$ muntazam piramidaning uchidagi yassi AQB burchagi α ga teng. Uning yon qirrasidagi ikki yoqli burchagini toping (12- rasm).

Yechish. Aytaylik, N nuqta AC qirraning o‘rtasi, AK esa A nuqtadan BQ qirraga tushurilgan perpendikular bo‘lsin.

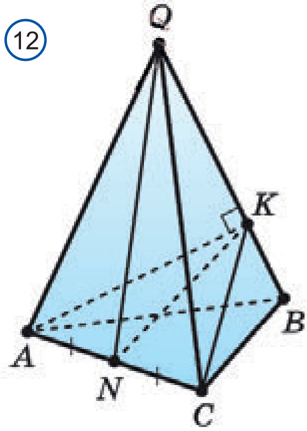
ABQ va CBQ uchburchaklarning tengligidan $CK \perp BQ$ bo‘ladi. Shuning uchun, AKC burchak BQ ikki yoqli burchakning chiziqli burchagi bo‘ladi.

AKQ va ANQ to‘g‘ri burchakli uchburchaklardan $AK = \sin\alpha$, $AN = AQ\sin(\alpha/2)$ ekanligini topamiz.

AKN to‘g‘ri burchakli uchburchaklardan esa $\sin\left(\frac{\angle AKC}{2}\right) = \frac{AN}{AK} = \frac{1}{2\cos(\alpha/2)}$ ga egamiz.

Bundan, $\angle AKC = 2\arcsin\frac{1}{2\cos(\alpha/2)}$. ❑

12



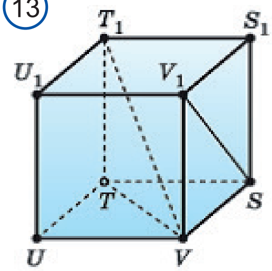
Mavzuga doir savollar va mashqlar

1. *Ikki yoqli burchak deb nimaga aytiladi?*
2. *Qanday burchak tekisliklar orasidagi burchak deb ataladi?*
3. *To‘g‘ri burchak ostida kesishuvchi tekisliklar qanday nomlanadi?*
4. *Tekisliklarning perpendikularlik alomatini ayting.*
5. *Perpendikular tekisliklarning xossalarini ayting va sharhlang.*

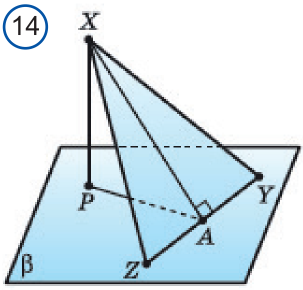
5.40. a) $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepipedning; b) $ABCA_1B_1C_1$ to‘g‘ri prizmaning perpendikular yoqlarini aniqlang va to‘g‘ri ikki yoqli burchaklarini ayting.

5.41. $STUVS_1T_1U_1V_1$ kubda (13- rasm): a) TVT_1 burchak; b) T_1ST burchak T_1SVT ikki yoqli burchakning chiziqli burchagi bo‘ladimi? V_1UTS ikki yoqli burchakning qiymatini toping.

13



14



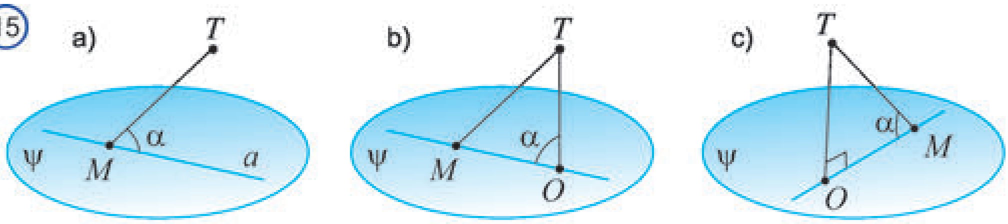
5.42. Ikkita ikki yoqli burchakning bittadan yog‘i umumiy, qolgan yoqlari birgalikda tekislikni tashkil qiladi. Bu ikki yoqli burchaklarning yig‘indisi 180° ga teng ekanligini isbotlang.

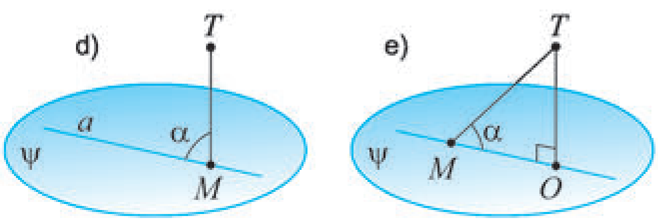
5.43. *XYZ* uchburchakning *YZ* tomoni β tekislikda yotadi. Uning *X* uchidan *XA* balandlik va β tekislikka *XP* perpendikular tushirilgan (14- rasm). *XAP* burchak *XYZP* ikki yoqli burchakning chiziqli burchagi ekanligini isbotlang.

5.44. Uchburchakli *ABCD* piramidaning *CD* qirrasi *ABC* tekislikka perpendikular. $AB = BC = AC = 6$ va $BD = 3\sqrt{7}$ bo‘lsa, *DACB, DABC, BDCA* ikki yoqli burchaklarni toping.

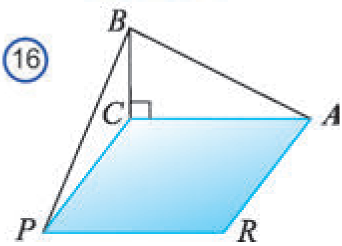
5.45. *T* nuqtadan ψ tekislikka og‘ma tushirilgan (15- rasm). Quyidagi rasmlarning qaysilarida tekislik va og‘ma orasidagi α burchak to‘g‘ri belgilangan?

15



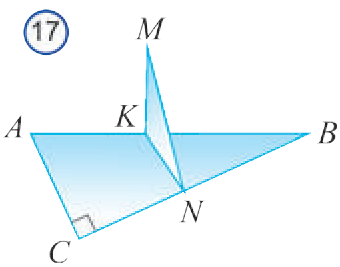


16



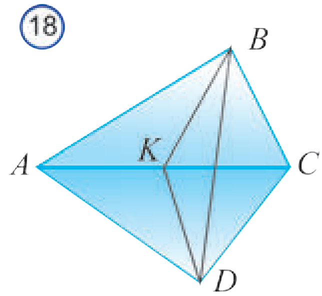
5.46. Uchburchakli *ABCD* piramidada *DAB, DAC, ACB* burchaklar to‘g‘ri, $AC = CB = 5$ va $DB = 5\sqrt{5}$ bo‘lsa, *ABCD* ikki yoqli burchagini toping.

17



5.47. Ikki yoqli burchak chiziqli burchagining tekisligi uning har bir yog‘ida perpendikular ekanligini isbotlang.

5.48. Ikki yoqli burchakning bitta yog‘ida yotgan ikkita nuqta uning qirrasidan, mos ravishda, 51 sm va 34 sm uzoqlikda yotibdi. Bu nuqtalarning birinchisi boshqa yog‘idan 15 sm uzoqlikda yotganligi ma'lum bo'lsa, shu yoqdan ikkinchi nuqtagacha bo'lgan masofani toping.



5.49. *ABC* to'g'ri burchakli uchburchak ($\angle C = 90°$) va *ACPR* kvadrat tekisliklari o'zaro perpendikular (16- rasm). Kvadrat tomoni 6 sm, uchburchak gipotenuzasi 10 sm. *BP* kesma uzunligini toping.

5.50. *MK* kesma to'g'ri burchakli *ABC* uchburchak ($\angle C = 90°$) tekisligiga perpendikular (17- rasm). $KN || AC$, $AK = KB$, $AC = 12$ sm, $MK = 8$ sm bo'lsa, *MN* kesma uzunligini toping.

5.51. *ABC* va *ADC* teng yonli uchburchaklar tekisliklari perpendikular (18- rasm). *AC* ularning umumiy asosi. *BK* kesma *ABC* uchburchak medianasi.

$BK = 8$ sm, $DK = 15$ sm bo'lsa, *BD* kesma uzunligini toping.

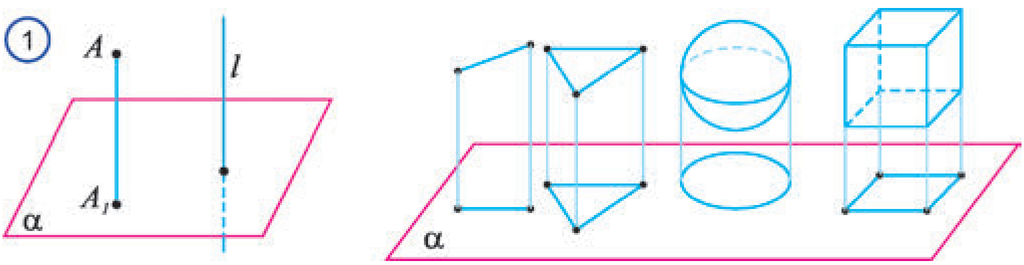
19 FAZODA ORTOGONAL PROYEKSIYA VA UNDAN TEXNIKADA FOYDALANISH

Agar proyeksiya yonalishi l proyeksiyalash tekisligi α ga perpendikular bo'lsa, bunday parallel proyeksiyalash *ortogonal proyeksiyalash* deb atalad (1- rasm).

Ortogonal proyeksiyalashda hosil bolgan shakl berilgan shaklning *ortogonal proyeksiyasi* yoki qisqacha *proyeksiyasi* deb aytiladi.

Parallel proyaeksiyalashning hamma xossalari ortogonal proyeksiyalashda ham o'rinli bo'ladi. Quyida faqat ortogonal proyeksiyaga tegishli bo'lgan muhim xossani isbotlaymiz.

5.13- teorema. Ko'pburchakning tekislikdagi ortogonal proyeksiyasining yuzi ko'pburchak yuzini uning tekisligi bilan proyeksiya tekisligi orasidagi burchak kosinusi ko'paytmasiga teng.

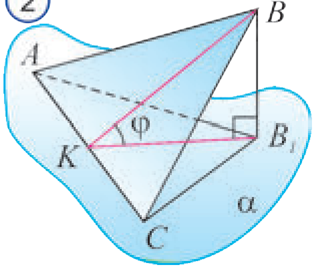


Isbot. 1) Avval uchburchak va uning biror tomonidan o'tuvchi tekislikdagi proyeksiyasi uchun qarab chiqamiz.

Aytaylik, *ABC* uchburchakning α tekislikdagi proyeksiyasi AB_1C uchburchak bo'lsin.

ABC uchburchaning *BK* balandligini tushiramiz. Uch perpendikularlar haqidagi

2



teoremaga ko‘ra, B_1K kesma KBB_1 uchburchakning balandligi bo‘ladi (2- rasm).

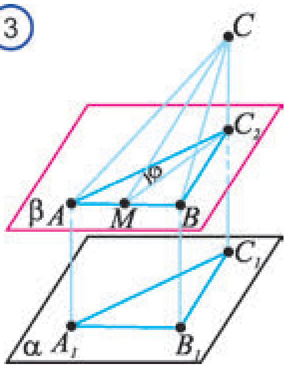
BKB_1 burchak – uchburchak tekisligi bilan proyeksiya tekisligi orasidagi φ burchakdan iborat bo‘ladi. BKB_1 uchburchakda: $KB_1 = KB \cdot \cos\varphi$.

U holda, $S_{ABC} = \frac{1}{2} AC \cdot KB$, $S_{AB_1C} = \frac{1}{2} AC \cdot KB_1$.

Bulardan, $S_{AB_1C} = S_{ABC} \cdot \cos\varphi$ ni hosil qilamiz. 1- holda teorema isbotlandi.

2) α tekislik o‘rniga unga parallel bo‘lgan boshqa β tekislik olinganda ham teorema o‘rinli bo‘ladi (3- rasm). Bu parallel proyeksiyalash xossasidan foydalanib isbotlanadi.

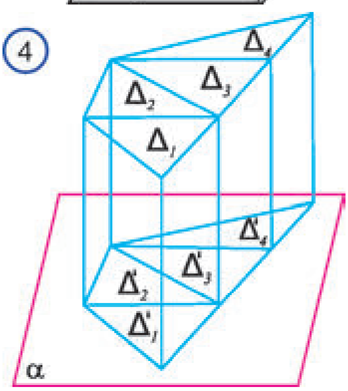
3



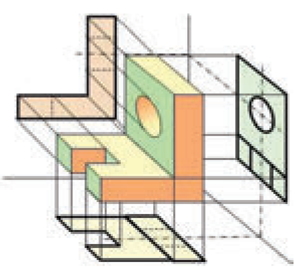
3) Endi umumiy, ko‘pburchak holiga keladigan bo‘lsak (4- rasm), bu holda teorema ko‘pburchakni diagonallari yordamida uchburchaklarga bo‘lish yordamida yuqorida ko‘rilgan xususiy holga keltirib isbotlanadi. ❑

Ortogonal proyeksiyadan texnik chizmachilikda turli xil detallarni loyihalashda foydalaniladi. Turli mashina detallari chizmalari bitta, ikkita yoki uchta o‘zaro perpendikular proyeksiyalar tekisliklariga ortogonal proyeksiyalash yo‘li bilan hosil qilinadi (5- rasmlar). Bu proyeksiyalar qaysi yo‘nalishda proyeksiyalanganligiga qarab, vertikal (tik), gorizontal va frontal proyeksiyalar deb ham ataladi.

4



5



? *Mavzuga doir savollar va mashqlar*

1. *Ortogonal proyeksiyalash deb nimaga aytiladi?*
2. *Ortogonal proyeksiyalash xossalarini sanang.*
3. *Ortogonal proyeksiyalashdan texnikada qanday foydalaniladi?*
4. *Bitta to‘g‘ri chiziqqa perpendikular bo‘lgan tekislik-ning xossasini ayting.*
5. *Umumlashgan Pifagor teoremasi nima haqida?*
6. *Uchunchi tekislikka prependikular ikki to‘g‘ri chiziq o‘zaro parallel bo‘ladimi?*
7. *Ikkinchi tekislikka perpendikular tekislik va to‘g‘ri chiziq o‘zaro parallel bo‘ladimi?*

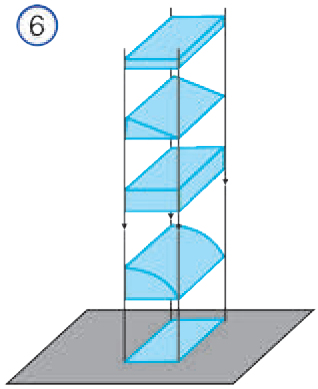
8. *Berilgan to'g'ri chiziqdan berilgan tekislikka perpendikular bo'lgan nechta tekislik o'tkazish mumkin?*
9. *α tekislik β tekislikka perpendikular. α tekislikdagi har qanday to'g'ri chiziq β tekislikka perpendikular bo'ladimi?*
10. *Birinchi tekislikka og'ma bo'lgan kesmadan o'tuvchi ikkinchi tekislik birinchisiga perpendikular bo'ladimi?*
11. *To'g'ri burchakli parallelepipedning kesishuvchi yoqlari o'zaro perpendikular bo'ladimi?*

5.52. Trapetsiyaning ortogonal proyeksiyasi: a) kvadrat; b) kesma; c) to'g'ri to'rtburchak; d) parallelogramm; e) trapetsiya bo'lishi mumkinmi?

5.53. 6-rasmga qarab ortogonal proyeksiyasi to'g'ri to'rtburchak bo'lgan geometrik shakllarni ayting.

5.54. A_1B_1 kesma AB kesmaning α tekislika ortogonal proyeksiyasi (7- rasm). Agar $AB = 20$ sm, $AC = 10$ sm, $A_1B_1 = 12$ sm bo'lsa, B_1C_1 kesma uzunligini toping.

6

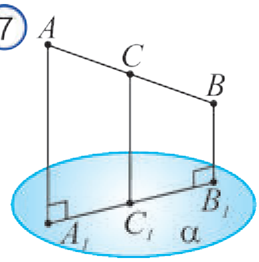


5.55. Uzunligi 5 sm bo'lgan AB kesmaning ω tekislikka ortogonal proyeksiyasi uzunligi 3 sm bo'lgan AC kesmadan iborat (8- rasm). AB kesmaning ω tekislika og'ish burchagi kosinusini toping.

5.56. Agar AB to'g'ri chiziqdan C nuqtagacha bo'lgan masofa (9- rasm) C nuqtadan ABD tekislikkacha bo'lgan masofadan ikki marta katta bo'lsa, ABC va ABD tekisliklar orasidagi burchakni toping.

5.57. ABC uchburchakning yuzi 18 sm^2 ga teng. $KC \perp (ABC)$. Agar ABK va ABC uchburchaklar tekisliklari orasidagi burchak: a) $\alpha = 30°$; b) $\alpha = 45°$; c) $\alpha = 60°$ bo'lsa, ABK uchburchak yuzini toping (10- rasm).

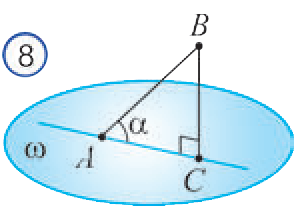
7



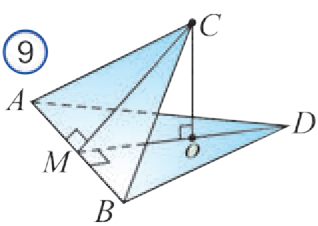
5.58. ABC va ABD uchburchaklar tekisliklari orasidagi burchak 60° ga teng. Agar $AB = 4\sqrt{3}$ bo'lsa, CD masofani toping.

5.59. Yuzi 48 sm^2 ga teng bo'lgan uchburchakning ortogonal proyeksiyasi tomonlari 14 sm, 16 sm va 6 sm bo'lgan uchburchakdan iborat. Bu uchburchak

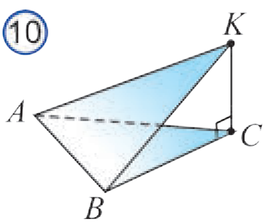
8



9



10



tekisligi va uning proyeksiyasi orasidagi burchakni hisoblang.

5.60. Yuzi 12 sm^2 ga teng bo‘lgan uchburchakning ortogonal proyeksiyasi - tomonlari 13 sm, 14 sm va 15 sm bo‘lgan uchburchakdan iborat. Bu uchburchak tekisligi va uning proyeksiyasi orasidagi burchakni hisoblang.

20 AMALIY MASHQ VA TATBIQ

Tatbiqlar va amaliy kompetensiyalarni shakllantirish

1. Ikki qo‘shni xona devorlari tutashgan chiziqning polga perpendikularligini qanday qilib o‘lchashlar yordamida tekshirsa bo‘ladi?

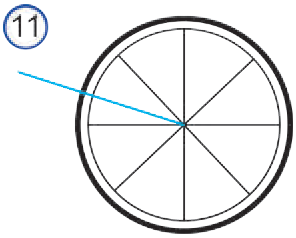
2. Uzunlik o‘lchov asbobi – puletka yordamida ustunning tikligini qanday tekshirsa bo‘ladi?

3. G‘ildirak o‘qi tekisligining u g‘ildirayotgan tekislikka perpendikularligini qanday tekshirsa bo‘ladi?

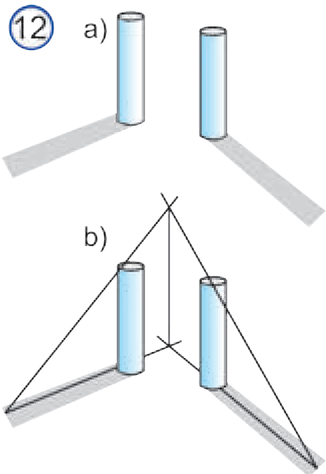
4. Nima sababdan qishda tomdan osilib turgan sumalaklarni, ularning qalinligini hisobga olmasdan, o‘zaro parallel deyish mumikn?

5. O‘quvchi amaliy ish bajaryapti. O‘rnatilgan bir necha ustunning Yerga nisbatan tikligini tekshirish uchun ulardan faqat bittasini tekshirdi. Qolgan ustunlarning tikligini quyidagicha tekshirdi: hamma ustunlarning balandligini, ularning pastki asoslari va yuqori uchlari orasidagi masofalarni o‘lchab qaror qabul qildi. U bu ishni to‘g‘ri bajardimi?

11



12



6. Nima sababdan eshik, u ochiqmi yoki yopiqmi har safar polga nisbatan perpendikular bo‘ladi?

7. To‘g‘ri chiziqning tekislikka perpendikularligiga yaqqol misol sifatida gildirak simlari yotgan tekislikning gildirak o‘qiga nisbatan bo‘lan joylashuvini keltirish mumkin (11- rasm). O‘q gildirakning har bir simiga perpenikular. Harakat davomida g‘ildirak simlari har biri bitta nuqtada kesishadigan kesmalardan iborat doira tekisligini hosil qiladi. Agar o‘q gorizontal joylashgan bo‘lsa, g‘ildirak qanday tekislikda aylanadi? Nega?

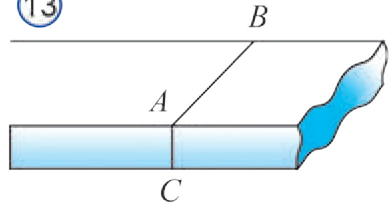
Ko‘rsatma: g‘ildirak o‘qiga perpendikular tekislikka perpendikular bo‘ladi.

8. Balandlikka sakrash mashqi bajarilmoqda. To‘siq tayoqni qo‘yish uchun qirrasi 25 m bo‘lgan kub va o‘lchamlri 25×25×50 bo‘lgan to‘g‘ri burchakli parallelepipedlardan foydalanilmoqda.

1) 125 sm; 2)150 sm; 3) 175 sm balandlikka sakrash mashqlarini qanday tashkil qilsa bo‘ladi?

9. 6- rasmda ikkita vertikal ustun va ularning soyasi tasvirlangan. Shu ma’lumotlardan foyadalanib, yorug‘lik manbasi (chiroq) joylashgan nuqtani va uning gorizontal tekislikka proyeksiyasini toping va quyidagi savollarga javob bering.

13



a) Ustunlarning vertikalligining ahamiyati bormi?

b) Soya tushayotgan tekislikning gorizontalligining ahamiyati bormi?

c) Rasmda berilgan ma’lumotlarning hammasi ham muhimmi?

Yechish. 12-rasmda tegishli yasashlar keltirilgan. Yorug‘lik manbasining joyini topishda ustunlarning yo‘nalishi ahamiyatga ega emas, lekin ularning vertikal ekanligi muhim hisoblanadi. Agar ustunlar vertikal va soya gorizontal tekislikka tushayotgan bo‘lsa, masalani yechish uchun rasmdagi bitta ustunning soyasini va ikkinchi ustundan tushayotgan soyaning yo‘nalishini bilish kifoya (13.*b- rasm*).

10. Dumaloq stolga tomoni a ga teng bo‘lgan kvadrat shaklidagi dasturxon solingan. Doira markazi kvadrat markazi bilan ustma-ust tushadi. Dasturxonning uchlari uning tomonlari o‘rtalariga nisbatan qanchalar polga yaqinroq?

Javob: $a(\sqrt{2}-1)/2 = 0{,}207\,a$.

11. Devorlarnng tikligini shoqul (bir uchiga tosh bog‘langan ip) bilan tekshiriladi. Agar shoqulning ipi devorga qanchalik yopishib tursa, devor shunchalik tik degan qarorga kelinadi. Bu qaror qanchalik to‘g‘ri? Bu tekshirish usuli nimaga asoslangan?

12. Arralash sirti arralanayotgan taxtaning hamma qirralariga perpendikular bo‘lishini ta’minlash uchun (13- rasm) taxta sirtida arralash chziqlarini qanday belgilash kerak?

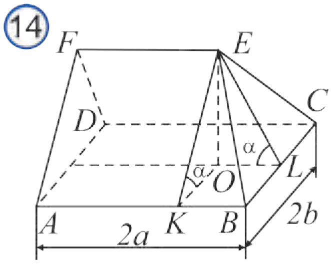
13. Xonaning qo‘shni devorlarining o‘zaro perpendikularligini tekshirish uchun Pifagor teoremasidan qanday foydalansa bo‘ladi?

14. Ustunning tikligini tekshirish uchun ustun asosi bilan bitta to‘g‘ri chiziqda yotmagan ikki nuqtadan kuzatiladi. Bunday tekshirish usulini asoslang.

Ko‘rsatma: To‘g‘ri chiziq va tekislikning perpendikularlik alomatidan foydalaning.

15. Borib bo‘lmaydigan tepalikdagi nuqtada baland ustun o‘rnatilgan. Shoqul yordamida uning tikligini qanday tekshirsa bo‘ladi?

Yechish: Ustunning biror vertikal to‘g‘ri chiziq bilan bitta tekislikda yotishini va yana boshqa vertikal to‘g‘ri chiziq bilan bitta (*boshqa*) *tekislikda yotishini*



ko‘rsatish yetarli bo‘ladi. Shoqulni oldimizga shunday qo‘yamiski, uning va ustunning yuqori uchlari hamda ko‘zimiz bitta to‘g‘ri chiziqda yotganda, shoqul ipi va ustun bitta to‘g‘ri chiziqda yotsin. Bu usul quyidagilarga asoslangan: 1) *vertikal ustun ixtiyoriy vertikal to‘g‘ri chiziq bilan bitta tekislikda yotishi kerak*; 2) *agar ikki parallel to‘g‘ri chiziq ikkita kesishuvchi tekislikda yotsa, bu to‘gri chiziqlar tekisliklarning kesishish chizig‘iga ham parallel bo‘ladi.*

16. Ikki vertikal joylashgan yassi oyna berilgan. Bu oynalardan birining sirtiga parallel bo‘lgan gorizontal nur ikkinchi oynadan birinchi oyna sirtiga perpendikular bo‘lgan to‘g‘ri chiziq bo‘yicha qaytadi. Oynalar orasidagi burchakni toping.

Ko‘rsatma: Yorug‘likning qaytish qonunidan foydalaning. *Javob*; 45°.

17. Gorizontal nur vertikal joylashgan ikki yassi oynadan qaytmoqda. Dastlab nur birinchi oyna sirtiga parallel bo‘lgan bo‘lsa, ikki marta akslanish natijasida ikkinchi oyna tekislgiga paralllel bo‘lib qolmoqda. Oynalar orasidagi burchakni toping.

Javob: 60°.

18. Qalinligi 5 m, yuzi 4 m^2 bo‘lgan, kvadrat shaklidagi po‘lat platforma to‘rtta uchidan tros sim bilan gorizontal osilgan. Har bir tros sim uzunligi 2 m. Tros simlarning platformaga nisbatan og‘ish burchagini topng. Balandligi 0,9 m, asosining diametri 0,6 m bo‘lgan silindr shaklidagi bakni bu platformaga joylashtirib bo‘ladimi?

Javob: 45°, *bakni joylashtirib bo‘ladi.*

19. Suv to‘rt tomonidan oqib tushadigan tom asosiga ortogonal proyeksiyalangan. Tom qirralarining proyeksiyasi to‘g‘ri to‘rtburchak shaklidagi tom asosi burchagining bissektrisasi bo‘lishini isbotlang.

20. Asosi *ABCD* to‘g‘ri to‘rtburchakdan iborat uyga yomg‘ir suvi to‘rt tomonidan oqib tushadigan tom o‘rnatish kerak (14- rasm). $AB = 2a$ m, $BC =$ $= 2b$ m. Tomning hamma yoqlari asos tekisligi bilan α burchak tashkil qiladi. Bu tomni yopish uchun qancha tunuka kerak bo‘ladi. Bunda tom sirti yuzining *k* foizi miqdoridagi tunuka chiqitga ketishini hisobga oling.

Javob: $4ab\,(1 + 0{,}01k\,)\,/\cos\alpha$.

21. Shamolsiz havoda yomg‘ir “qiyalab” yog‘moqda. To‘g‘ri to‘rtburchak shaklidagi faner bo‘lagi yordamida yomg‘rning gorizont tekisligiga nisbatan qiyaligini qanday aniqlasa bo‘ladi? Tegishli chizmani chizing.

Ko‘rsatma: Faner bo‘lagini shunday joylashtirish kerakki, uning tekisligi *yomg'ir tomchilari harakat trayektoriyasi va ularning gorizontal tekislikka proyeksiyasi aniqlagan tekislikka taxminan perpendikular bo'lsin. Shunda, gorizontal tekislikda yomg'ir tushmaydigan to'g'ri to'rtburchak hosil bo'ladi. So'ng tegishli kesmalarning uzunliklari o'lchanadi va ular orasidagi burchakning tangensi hisoblanadi.*

22. Yuzi S_1 ga, uzunligi n ga teng bo'lgan bolalar krovati ustini ikkita bir xil to'g'ri to'rtburchak shaklidagi pardalar bilan yopish kerak. Har bir pardaning yuzi S_2 ga, uzunligi esa krovat uzunligiga teng. Har ikkala pardaning yuqori cheti krovat ustida parallel o'rnatilgan va krovat uzunligiga teng simga mahkamlangan. Simning krovatdan qanday balandlikda o'rnatilganligini toping. Masalani quyidagi sonli shartlarda yeching: $n = 1$ m 20 sm, $S_1 = 6000$ sm², $S_2 = 7800$ sm². Tegishli chizmani chizing. *Ko'rsatma:* $\sqrt{4S_2^2 - S_1^2} / 2n$.

Javob: 0,5 m.

23. Asosi $ABCD$ to'g'ri to'rtburchakdan iborat uyga yomg'ir suvi to'rt tomonidan oqib tushadigan tom o'rnatish kerak (8- rasm). $AB = 18$ m, $BC = 12$ m. Tomning hamma yoqlari asos tekisligi bilan 40° li burchak tashkil qiladi. Agar 1 m² yuzani yopish uchun 15 dona cherepitsa ishlatilsa, bu tomni yopish uchun necha dona cherepitsa kerak bo'ladi?

24. Oltiyoqli qalam va ochilgan kitob yordamida to'g'ri chiziqlar orasidagi, to'g'ri chiziq va tekislik orasidagi, tekisliklar orasidagi burchaklarning timsollarini ko'rsating.

25. 14-rasmda tasvirlangan ikkita simmertiya o'qiga ega, tomdan yomg'ir suvi qaysi yo'nalishlarda oqib tushishini aniqlang.

26. Asosiga borib bo'lmaydigan minoraning balandligini aniqlash uchun qanday o'lchashlarni amalga oshirish kerak?

27. Balandligi ma'lum, lekin yaqiniga borib bo'lmaydigan binogacha bo'lgan masofani topish uchun qanday o'lchashlarni amalga oshirish kerak?

28. Nega soyalar choshgohda (tushda) yo'qoladi?

29. Daraxt tepasiga chiqmasdan uning balandligini qanday o'lchasa bo'ladi?

Javoblar

4.5. a) 7 sm; b) 30 sm. **4.6.** b) 200 sm. **4.13.** 50 sm. **4.14.** 40 mm. **4.21.** $a + b$. **4.22.** a) 40°; b) 45°; c) 90°. **4.23.** a)58°; b) 47°. **4.51.** 32 sm. **4.52.** 6 sm. **4.53.** 20 sm.

5.11. 1) 6,5 sm; 2) 15 sm; 3) $\sqrt{2a^2 - b^2 + d^2}$; 3) $\sqrt{2a^2 - c^2 + 2d^2}$. **5.12.** 2 m. **5.17.** 15 sm va 41 sm. **5.20.** $BD = \sqrt{2a^2 + b^2 + c^2}$; $CD = \sqrt{a^2 + c^2}$. **5.21.** 3,9 m. **5.22.** 9 m. **5.23.** a) $\sqrt{2}/2$; b) $\sqrt{(5 + 3\cos\beta)/2}$. **5.24.** 3 sm; 7,5 sm. **5.25.** 20 sm. **5.34.** $3d$. **5.37.** 45°. **5.38.** $\arccos\sqrt{3}/3$. **5.44.** 90°. **5.46.** 60°.

M.A. Mirzaahmedov, Sh.N. Ismailov, A.Q. Amanov, B.Q. Haydarov

MATEMATIKA 10
ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
II QISM

O‘rta ta’lim muassasalarining 10-sinfi va o‘rta maxsus, kasb-hunar ta’limi muassasalari o‘quvchilari uchun darslik
1- nashr

Muharrir:	N. Gayipov
Texn. Muharrir:	K. Madiarov
Kompyuterda sahifalovchi:	S.G‘ofurov

Nashriyot litsenziyasi AI № 296. 22.05.2017
Bosishga ruxsat etildi 29.11.2017. Bichimi 70×100 $^{1}/_{16}$
«TimesNewRoman» garniturasi.
Hajmi: 9,0 bosma tab. Nashr tab. 9,0.
Adadi 428121 nusxada

Original-maket «Extremum-press» MCHJda tayyorlandi. 100053, Toshkent sh. Bog‘ishamol ko‘chasi, 3. Tel: 234-44-05

O‘zbekiston matbuot va axborot agentligining «O‘qituvchi» nashriyot-matbaa ijodiyot uyi bosmaxonasida chop etildi.
100206, Toshkent sh. Yunusobod dahasi,
Yangishahar ko‘chasi, 1- uy.
Buyurtma № .